Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/32-1.2.1.1-a+b-x+c-x^2-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [143]. This is test number [32].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (143)	0.00 (0)
Mathematica	100.00 (143)	0.00 (0)
Mupad	92.31 (132)	7.69 (11)
Fricas	79.02 (113)	20.98 (30)
Maple	79.02 (113)	20.98 (30)
Giac	77.62 (111)	22.38 (32)
Maxima	75.52 (108)	24.48 (35)
Sympy	32.87 (47)	67.13 (96)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

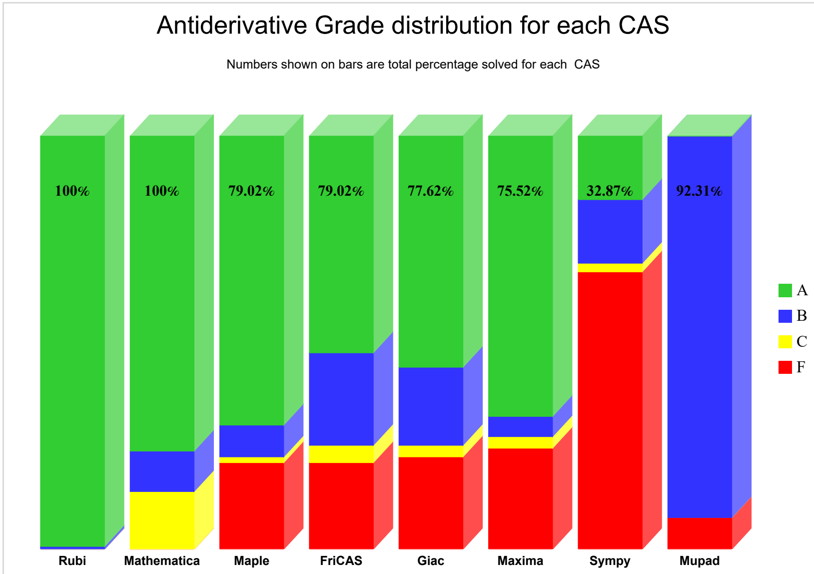
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

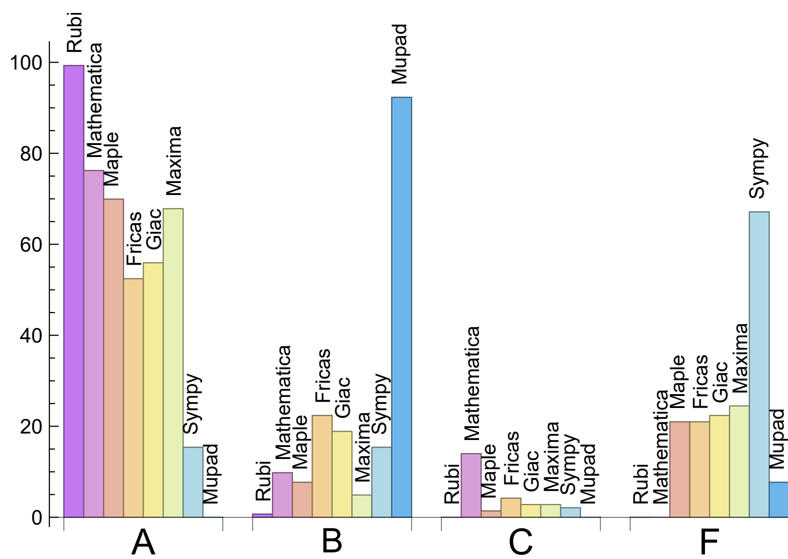
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.30	0.70	0.00	0.00
Mathematica	76.22	9.79	13.99	0.00
Maple	69.93	7.69	1.40	20.98
Maxima	67.83	4.90	2.80	24.48
Giac	55.94	18.88	2.80	22.38
Fricas	52.45	22.38	4.20	20.98
Sympy	15.38	15.38	2.10	67.13
Mupad	N/A	92.31	0.00	7.69

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	30	100.00 %	0.00 %	0.00 %
Fricas	30	100.00 %	0.00 %	0.00 %
Giac	32	100.00 %	0.00 %	0.00 %
Maxima	35	85.71 %	0.00 %	14.29 %
Sympy	96	100.00 %	0.00 %	0.00 %
Mupad	11	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

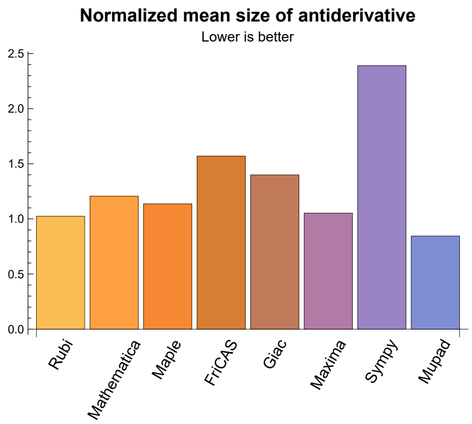
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

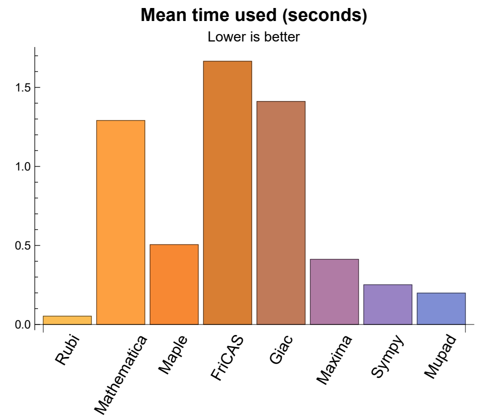
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.05	83.41	1.02	38.00	1.00
Mathematica	1.29	49.15	1.21	43.00	1.00
Maple	0.51	56.72	1.14	32.00	0.86
Maxima	0.41	45.49	1.05	33.50	1.03
Fricas	1.67	63.38	1.57	43.00	1.21
Sympy	0.25	118.68	2.39	70.00	1.81
Giac	1.41	54.23	1.40	37.00	1.05
Mupad	0.20	39.27	0.84	33.50	0.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {102}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

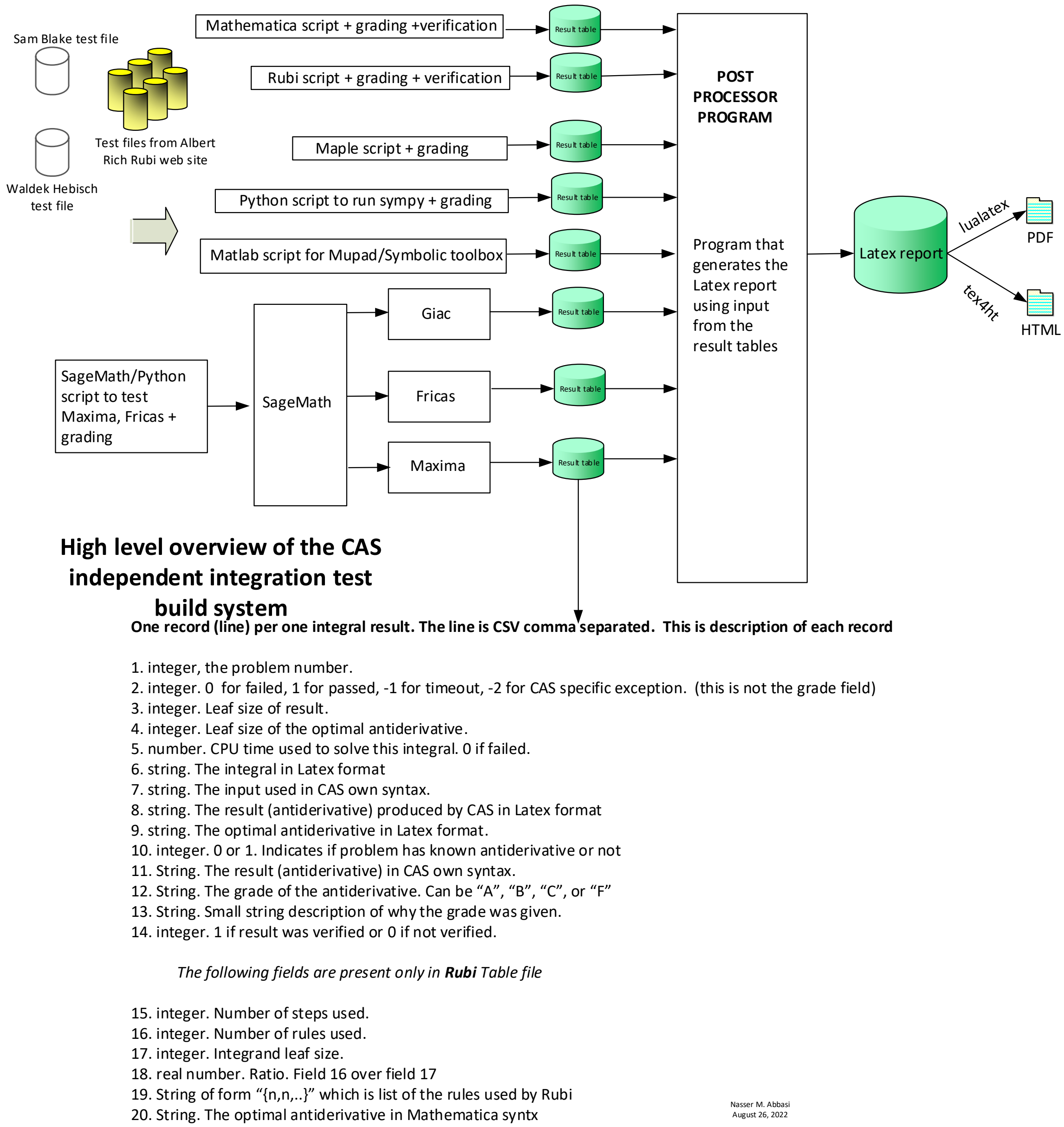
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { 83 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 116, 117, 119, 120, 121, 123, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { 17, 21, 25, 26, 27, 28, 29, 83, 102, 115, 118, 124, 126, 127 }

C grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 111, 122 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 129, 130, 131, 138 }

B grade: { 25, 74, 75, 76, 77, 83, 101, 102, 125, 126, 127 }

C grade: { 65, 68 }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 78, 79, 80, 81, 82, 84, 85, 86, 87, 90, 91, 92, 93, 94, 97, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 138 }

B grade: { 17, 74, 75, 76, 77, 83, 101 }

C grade: { 71, 73, 111, 122 }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 88, 89, 95, 96, 98, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 22, 23, 24, 26, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 78, 79, 81, 82, 86, 88, 89, 91, 92, 93, 94, 98, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 116, 119, 121, 123, 130, 131, 138 }

B grade: { 17, 18, 21, 25, 27, 74, 75, 76, 77, 80, 83, 84, 85, 87, 90, 95, 96, 97, 99, 100, 104, 114, 115, 117, 118, 120, 124, 125, 126, 127, 128, 129 }

C grade: { 70, 71, 72, 73, 111, 122 }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.6 Sympy

A grade: { 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 68, 69, 78, 79, 80, 81, 82, 91, 92, 93, 94, 138 }

B grade: { 53, 54, 61, 62, 63, 74, 75, 76, 77, 83, 84, 85, 87, 88, 89, 90, 95, 96, 97, 98, 99, 100 }

C grade: { 86, 101, 102 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 64, 65, 66, 67, 70, 71, 72, 73, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.7 Giac

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 22, 23, 24, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 116, 119, 121, 123, 128, 129, 130, 131, 138 }

B grade: { 3, 4, 5, 17, 18, 21, 25, 26, 27, 28, 29, 64, 74, 75, 76, 77, 83, 101, 114, 115, 117, 118, 120, 124, 125, 126, 127 }

C grade: { 70, 71, 72, 73 }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 111, 122, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 138 }

C grade: { }

F grade: { 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	157	149	180	258	0	132	151
N.S.	1	1.00	1.07	1.01	1.22	1.76	0.00	0.90	1.03
time (sec)	N/A	0.036	0.178	0.400	0.282	1.297	0.000	2.464	0.735

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	117	91	130	69	0	140	100
N.S.	1	1.00	0.97	0.75	1.07	0.57	0.00	1.16	0.83
time (sec)	N/A	0.020	0.118	0.555	0.538	1.721	0.000	1.260	0.292

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	86	71	103	59	0	130	80
N.S.	1	1.00	0.91	0.75	1.08	0.62	0.00	1.37	0.84
time (sec)	N/A	0.015	0.093	0.409	0.534	1.310	0.000	2.635	0.298

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	83	51	76	49	0	120	60
N.S.	1	1.00	1.20	0.74	1.10	0.71	0.00	1.74	0.87
time (sec)	N/A	0.010	0.077	0.401	0.544	1.565	0.000	1.943	0.162

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	31	49	39	0	110	39
N.S.	1	1.00	1.44	0.72	1.14	0.91	0.00	2.56	0.91
time (sec)	N/A	0.007	0.048	0.457	0.520	1.438	0.000	2.613	0.087

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	82	117	68	0	57	81
N.S.	1	1.00	1.01	0.81	1.16	0.67	0.00	0.56	0.80
time (sec)	N/A	0.018	0.103	0.418	0.475	1.514	0.000	1.381	0.166

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	92	64	90	58	0	47	63
N.S.	1	1.00	1.16	0.81	1.14	0.73	0.00	0.59	0.80
time (sec)	N/A	0.013	0.076	0.414	0.479	1.383	0.000	2.010	0.242

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	82	46	63	48	0	37	45
N.S.	1	1.00	1.44	0.81	1.11	0.84	0.00	0.65	0.79
time (sec)	N/A	0.009	0.077	0.413	0.519	1.632	0.000	1.805	0.112

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	55	28	36	38	0	27	26
N.S.	1	1.00	1.57	0.80	1.03	1.09	0.00	0.77	0.74
time (sec)	N/A	0.006	0.046	0.408	0.523	1.599	0.000	2.933	0.052

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	28	36	35	0	25	26
N.S.	1	1.00	1.23	0.80	1.03	1.00	0.00	0.71	0.74
time (sec)	N/A	0.007	0.055	0.402	0.552	1.307	0.000	1.581	0.048

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	55	28	36	38	0	27	26
N.S.	1	1.00	1.57	0.80	1.03	1.09	0.00	0.77	0.74
time (sec)	N/A	0.006	0.050	0.400	0.522	1.513	0.000	1.682	0.054

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	67	42	55	43	0	35	39
N.S.	1	1.00	1.31	0.82	1.08	0.84	0.00	0.69	0.76
time (sec)	N/A	0.007	0.131	0.409	0.528	1.423	0.000	1.915	0.194

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	42	33	41	32	0	33	29
N.S.	1	1.00	1.20	0.94	1.17	0.91	0.00	0.94	0.83
time (sec)	N/A	0.005	0.050	0.393	0.296	1.717	0.000	2.536	0.199

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	33	43	32	0	33	29
N.S.	1	1.00	1.14	0.89	1.16	0.86	0.00	0.89	0.78
time (sec)	N/A	0.005	0.054	0.409	0.278	1.839	0.000	1.837	0.109

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	33	43	36	0	37	29
N.S.	1	1.00	1.28	0.85	1.10	0.92	0.00	0.95	0.74
time (sec)	N/A	0.005	0.077	0.407	0.274	1.595	0.000	2.067	0.195

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	76	111	105	0	74	96
N.S.	1	1.00	0.84	0.92	1.34	1.27	0.00	0.89	1.16
time (sec)	N/A	0.012	0.123	0.421	0.294	1.962	0.000	1.093	0.271

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	51	10	21	19	0	110	19
N.S.	1	1.00	3.19	0.62	1.31	1.19	0.00	6.88	1.19
time (sec)	N/A	0.005	0.029	0.432	0.492	1.346	0.000	1.582	0.276

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	28	39	0	64	20
N.S.	1	1.00	0.92	0.81	1.08	1.50	0.00	2.46	0.77
time (sec)	N/A	0.002	0.049	0.378	0.264	2.004	0.000	1.897	0.045

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	42	55	63	0	74	31
N.S.	1	1.00	0.68	0.79	1.04	1.19	0.00	1.40	0.58
time (sec)	N/A	0.005	0.068	0.403	0.263	1.441	0.000	2.097	0.124

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	48	62	82	83	0	84	40
N.S.	1	1.00	0.61	0.78	1.04	1.05	0.00	1.06	0.51
time (sec)	N/A	0.008	0.093	0.421	0.295	1.650	0.000	1.778	0.286

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	46	9	8	19	0	27	8
N.S.	1	1.00	3.83	0.75	0.67	1.58	0.00	2.25	0.67
time (sec)	N/A	0.004	0.030	0.386	0.539	1.329	0.000	3.702	0.106

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	19	28	29	0	29	18
N.S.	1	1.00	0.95	0.86	1.27	1.32	0.00	1.32	0.82
time (sec)	N/A	0.002	0.048	0.376	0.291	1.655	0.000	1.666	0.136

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	38	55	46	0	39	28
N.S.	1	1.00	0.69	0.84	1.22	1.02	0.00	0.87	0.62
time (sec)	N/A	0.004	0.064	0.388	0.309	1.384	0.000	2.579	0.032

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	41	56	82	61	0	49	73
N.S.	1	1.00	0.61	0.84	1.22	0.91	0.00	0.73	1.09
time (sec)	N/A	0.008	0.081	0.380	0.305	1.762	0.000	1.824	0.195

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	57	35	21	27	0	41	42
N.S.	1	1.00	4.75	2.92	1.75	2.25	0.00	3.42	3.50
time (sec)	N/A	0.006	0.044	0.378	0.478	1.270	0.000	2.727	0.312

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	56	37	29	27	0	59	36
N.S.	1	1.00	2.33	1.54	1.21	1.12	0.00	2.46	1.50
time (sec)	N/A	0.005	0.041	0.371	0.303	1.561	0.000	2.220	0.226

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	38	7	8	18	0	25	6
N.S.	1	1.00	3.80	0.70	0.80	1.80	0.00	2.50	0.60
time (sec)	N/A	0.004	0.028	0.379	0.501	1.663	0.000	2.837	0.109

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	37	14	17	17	0	33	11
N.S.	1	1.00	2.31	0.88	1.06	1.06	0.00	2.06	0.69
time (sec)	N/A	0.003	0.026	0.367	0.267	1.455	0.000	1.970	0.511

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	37	14	17	17	0	33	11
N.S.	1	1.00	2.31	0.88	1.06	1.06	0.00	2.06	0.69
time (sec)	N/A	0.003	0.028	0.441	0.271	1.606	0.000	1.949	0.529

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	48	0	0	0	0	0	36
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	0.08
time (sec)	N/A	0.501	10.025	0.044	0.000	0.000	0.000	0.000	0.234

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	45	0	0	0	0	0	36
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.353	9.391	0.044	0.000	0.000	0.000	0.000	0.173

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	43	0	0	0	0	0	36
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	0.11
time (sec)	N/A	0.312	9.277	0.100	0.000	0.000	0.000	0.000	0.215

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	47	0	0	0	0	0	36
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.353	10.019	0.043	0.000	0.000	0.000	0.000	0.249

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	50	0	0	0	0	0	36
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	0.08
time (sec)	N/A	0.399	10.016	0.046	0.000	0.000	0.000	0.000	0.275

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	842	842	48	0	0	0	0	0	36
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	0.804	10.025	0.046	0.000	0.000	0.000	0.000	0.181

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	781	781	45	0	0	0	0	0	36
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.655	10.016	0.043	0.000	0.000	0.000	0.000	0.160

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	715	715	45	0	0	0	0	0	36
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.599	9.591	0.096	0.000	0.000	0.000	0.000	0.200

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	773	773	45	0	0	0	0	0	36
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.682	10.016	0.045	0.000	0.000	0.000	0.000	0.231

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	838	838	50	0	0	0	0	0	36
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	1.124	10.027	0.049	0.000	0.000	0.000	0.000	0.245

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	48	0	0	0	0	0	36
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	0.30
time (sec)	N/A	0.032	10.020	0.042	0.000	0.000	0.000	0.000	0.190

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	45	0	0	0	0	0	36
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.022	10.014	0.042	0.000	0.000	0.000	0.000	0.169

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	45	0	0	0	0	0	36
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.022	10.011	0.041	0.000	0.000	0.000	0.000	0.172

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	45	0	0	0	0	0	36
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.62
time (sec)	N/A	0.015	10.020	0.088	0.000	0.000	0.000	0.000	0.199

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	0	0	0	0	0	36
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.61
time (sec)	N/A	0.015	10.012	0.089	0.000	0.000	0.000	0.000	0.189

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	45	0	0	0	0	0	36
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	0.43
time (sec)	N/A	0.021	10.026	0.041	0.000	0.000	0.000	0.000	0.224

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	50	0	0	0	0	0	36
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	0.31
time (sec)	N/A	0.030	10.018	0.044	0.000	0.000	0.000	0.000	0.256

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	50	0	0	0	0	0	36
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	0.25
time (sec)	N/A	0.041	10.017	0.044	0.000	0.000	0.000	0.000	0.288

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	45	0	0	0	0	0	48
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.007	0.024	0.053	0.000	0.000	0.000	0.000	0.316

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	43	43	49	43	43
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84
time (sec)	N/A	0.013	0.002	0.409	0.279	1.696	0.008	0.990	0.026

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.008	0.001	0.362	0.317	1.485	0.007	0.997	0.039

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.005	0.001	0.365	0.265	1.489	0.006	0.873	0.029

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.000	0.018	0.293	1.572	0.006	1.346	0.017

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.004	0.004	0.396	0.522	1.754	0.045	1.439	0.066

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.007	0.019	0.388	0.497	1.508	0.087	0.999	0.143

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	57	58	188	105	45	55
N.S.	1	1.00	0.89	0.92	0.94	3.03	1.69	0.73	0.89
time (sec)	N/A	0.011	0.030	0.386	0.519	1.907	0.145	1.605	0.162

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	68	58	146	97	63	37
N.S.	1	1.00	0.85	0.81	0.69	1.74	1.15	0.75	0.44
time (sec)	N/A	0.015	0.082	0.382	0.277	1.361	2.563	1.511	0.204

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	52	43	124	70	49	37
N.S.	1	1.00	0.92	0.80	0.66	1.91	1.08	0.75	0.57
time (sec)	N/A	0.010	0.055	0.375	0.297	1.586	1.561	1.207	0.157

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	36	28	94	41	37	35
N.S.	1	1.00	1.04	0.78	0.61	2.04	0.89	0.80	0.76
time (sec)	N/A	0.006	0.033	0.375	0.284	1.405	0.920	1.926	0.127

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	0.80
time (sec)	N/A	0.004	0.005	0.373	0.295	1.681	0.437	1.323	0.187

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88
time (sec)	N/A	0.001	0.028	0.369	0.267	2.098	0.352	1.331	0.032

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	32	31	47	95	27	28
N.S.	1	1.00	0.74	0.82	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.004	0.045	0.373	0.269	1.655	0.433	1.987	0.192

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	53	46	69	413	41	44
N.S.	1	1.00	0.69	0.91	0.79	1.19	7.12	0.71	0.76
time (sec)	N/A	0.007	0.066	0.431	0.284	2.262	0.734	1.024	0.193

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	74	61	91	1265	55	61
N.S.	1	1.00	0.66	0.96	0.79	1.18	16.43	0.71	0.79
time (sec)	N/A	0.011	0.072	0.407	0.291	1.561	1.040	0.843	0.205

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	17	30	19	0	45	19
N.S.	1	1.00	0.87	0.74	1.30	0.83	0.00	1.96	0.83
time (sec)	N/A	0.002	0.008	0.434	0.489	2.150	0.000	2.437	0.049

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	16	30	9	0	26	19
N.S.	1	1.00	1.09	0.70	1.30	0.39	0.00	1.13	0.83
time (sec)	N/A	0.002	0.004	0.403	0.509	0.838	0.000	1.641	0.046

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	6	8	0	25	14
N.S.	1	1.00	0.90	0.79	0.21	0.28	0.00	0.86	0.48
time (sec)	N/A	0.004	0.004	0.447	0.480	1.499	0.000	1.417	0.283

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	17	9	14	0	17	21
N.S.	1	1.00	0.80	0.68	0.36	0.56	0.00	0.68	0.84
time (sec)	N/A	0.002	0.004	0.431	0.509	2.007	0.000	1.485	0.170

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	16	30	9	8	26	13
N.S.	1	1.00	1.09	0.70	1.30	0.39	0.35	1.13	0.57
time (sec)	N/A	0.002	0.008	0.072	0.506	1.383	0.008	1.344	0.113

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	6	8	7	15	14
N.S.	1	1.00	0.90	0.79	0.21	0.28	0.24	0.52	0.48
time (sec)	N/A	0.003	0.007	0.374	0.484	1.657	0.008	1.502	0.353

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	30	9	0	26	18
N.S.	1	1.00	1.17	1.17	1.30	0.39	0.00	1.13	0.78
time (sec)	N/A	0.002	0.005	0.519	0.484	1.449	0.000	0.985	0.327

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	6	6	0	23	15
N.S.	1	1.00	0.97	0.86	0.21	0.21	0.00	0.79	0.52
time (sec)	N/A	0.003	0.005	0.443	0.505	1.455	0.000	0.774	0.245

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	30	9	0	26	18
N.S.	1	1.00	1.17	1.17	1.30	0.39	0.00	1.13	0.78
time (sec)	N/A	0.002	0.004	0.403	0.476	1.318	0.000	0.831	0.067

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	6	6	0	23	15
N.S.	1	1.00	0.97	0.86	0.21	0.21	0.00	0.79	0.52
time (sec)	N/A	0.004	0.003	0.428	0.500	1.389	0.000	0.668	0.295

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	234	233	253	334	184
N.S.	1	1.00	1.90	5.83	2.15	2.14	2.32	3.06	1.69
time (sec)	N/A	0.102	0.024	0.620	0.295	1.466	0.075	0.646	0.314

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	234	235	250	334	184
N.S.	1	1.00	1.90	5.83	2.15	2.16	2.29	3.06	1.69
time (sec)	N/A	0.098	0.032	0.534	0.283	1.050	0.069	1.374	0.331

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	199	636	234	227	253	334	176
N.S.	1	1.00	1.83	5.83	2.15	2.08	2.32	3.06	1.61
time (sec)	N/A	0.103	0.022	0.494	0.286	1.774	0.070	1.790	0.337

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	234	235	248	334	184
N.S.	1	1.00	1.90	5.83	2.15	2.16	2.28	3.06	1.69
time (sec)	N/A	0.101	0.033	0.483	0.281	1.123	0.068	1.327	0.316

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	22	16	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	1.22	0.89	0.89
time (sec)	N/A	0.008	0.005	0.823	0.508	1.344	0.046	1.593	0.139

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	10	7	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.83	0.58	0.67	0.67
time (sec)	N/A	0.006	0.011	1.048	0.487	1.334	0.066	1.020	0.272

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	34	17	27	39	39	31	15
N.S.	1	1.00	1.79	0.89	1.42	2.05	2.05	1.63	0.79
time (sec)	N/A	0.012	0.014	0.549	0.512	1.982	0.040	0.837	0.209

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	8
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	0.62
time (sec)	N/A	0.003	0.003	0.449	0.281	1.315	0.028	0.626	0.082

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	15	14	17	8
N.S.	1	1.00	1.00	0.76	0.71	0.71	0.67	0.81	0.38
time (sec)	N/A	0.004	0.003	0.470	0.276	1.697	0.034	0.715	0.091

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	17	17	14	13	13	12	15	6
N.S.	1	2.83	2.83	2.33	2.17	2.17	2.00	2.50	1.00
time (sec)	N/A	0.003	0.002	0.556	0.287	1.387	0.028	0.642	0.176

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	38	50	76	40	23
N.S.	1	1.00	1.00	0.89	1.41	1.85	2.81	1.48	0.85
time (sec)	N/A	0.013	0.006	0.519	0.277	1.916	0.098	0.707	0.359

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	39	51	76	45	23
N.S.	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	0.85
time (sec)	N/A	0.013	0.006	0.581	0.267	0.969	0.124	0.680	0.386

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	41	87	27	23
N.S.	1	1.00	1.00	0.90	0.87	1.32	2.81	0.87	0.74
time (sec)	N/A	0.014	0.006	0.997	0.321	2.795	0.078	0.817	0.381

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	39	51	76	45	23
N.S.	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	0.85
time (sec)	N/A	0.013	0.007	0.590	0.289	1.941	0.131	0.712	0.418

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	0	113	124	34	46
N.S.	1	1.00	1.00	0.92	0.00	2.97	3.26	0.89	1.21
time (sec)	N/A	0.025	0.008	0.641	0.000	1.403	0.090	1.297	0.229

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	35	0	124	100	30	33
N.S.	1	1.00	0.97	1.00	0.00	3.54	2.86	0.86	0.94
time (sec)	N/A	0.019	0.007	0.638	0.000	1.350	0.106	1.458	0.273

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	32	49	67	102	55	28
N.S.	1	1.00	1.28	1.00	1.53	2.09	3.19	1.72	0.88
time (sec)	N/A	0.017	0.008	0.596	0.517	1.709	0.107	1.418	0.227

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	36	45	39	36	33
N.S.	1	1.00	1.00	0.86	0.84	1.05	0.91	0.84	0.77
time (sec)	N/A	0.010	0.017	0.635	0.508	1.188	0.048	0.856	0.038

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	37	47	68	58	51	34
N.S.	1	1.00	1.44	0.86	1.09	1.58	1.35	1.19	0.79
time (sec)	N/A	0.011	0.020	0.556	0.491	1.711	0.068	0.644	0.163

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	32	34	53	31	36	34
N.S.	1	1.00	0.97	0.94	1.00	1.56	0.91	1.06	1.00
time (sec)	N/A	0.006	0.009	0.607	0.274	1.351	0.046	0.696	0.200

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	34	53	32	36	34
N.S.	1	1.00	1.00	0.76	0.81	1.26	0.76	0.86	0.81
time (sec)	N/A	0.006	0.010	0.534	0.276	2.288	0.057	0.660	0.080

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	68	0	334	265	67	119
N.S.	1	1.00	0.99	0.96	0.00	4.70	3.73	0.94	1.68
time (sec)	N/A	0.028	0.043	0.606	0.000	2.167	0.303	0.831	0.171

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	86	0	317	230	75	107
N.S.	1	1.00	1.00	1.19	0.00	4.40	3.19	1.04	1.49
time (sec)	N/A	0.025	0.034	0.581	0.000	2.748	0.296	0.712	0.320

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	83	97	171	218	90	100
N.S.	1	1.00	1.13	1.20	1.41	2.48	3.16	1.30	1.45
time (sec)	N/A	0.025	0.045	0.579	0.516	3.386	0.293	0.913	0.316

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	111	0	89	212	100	110
N.S.	1	1.00	1.05	1.79	0.00	1.44	3.42	1.61	1.77
time (sec)	N/A	0.116	0.068	1.615	0.000	2.076	0.435	0.817	0.253

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	58	34	31	55	63	56	56	38
N.S.	1	1.76	1.03	0.94	1.67	1.91	1.70	1.70	1.15
time (sec)	N/A	0.024	0.022	0.600	0.270	1.255	0.127	1.054	0.258

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	58	32	31	51	59	56	54	38
N.S.	1	1.87	1.03	1.00	1.65	1.90	1.81	1.74	1.23
time (sec)	N/A	0.021	0.020	0.525	0.284	1.856	0.127	0.883	0.135

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	33	27	15	165	27	27
N.S.	1	1.00	1.12	1.94	1.59	0.88	9.71	1.59	1.59
time (sec)	N/A	0.014	0.015	1.091	0.472	0.840	0.077	1.667	0.117

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	56	39	33	21	70	33	42
N.S.	1	1.00	2.43	1.70	1.43	0.91	3.04	1.43	1.83
time (sec)	N/A	0.020	0.028	0.941	0.514	1.874	0.313	1.605	0.303

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	48	29	38	40	0	40	39
N.S.	1	1.00	1.26	0.76	1.00	1.05	0.00	1.05	1.03
time (sec)	N/A	0.008	0.056	0.701	0.539	1.100	0.000	1.288	0.080

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	49	25	38	53	0	24	23
N.S.	1	1.00	1.63	0.83	1.27	1.77	0.00	0.80	0.77
time (sec)	N/A	0.007	0.060	0.573	0.509	1.676	0.000	1.388	0.052

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	52	40	0	41	39
N.S.	1	1.00	1.00	1.02	1.06	0.82	0.00	0.84	0.80
time (sec)	N/A	0.008	0.068	0.489	0.544	1.257	0.000	1.457	0.210

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	53	35	46	58	0	53	48
N.S.	1	1.00	1.18	0.78	1.02	1.29	0.00	1.18	1.07
time (sec)	N/A	0.010	0.084	0.611	0.515	1.551	0.000	1.008	0.191

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	61	35	46	60	0	36	35
N.S.	1	1.00	1.36	0.78	1.02	1.33	0.00	0.80	0.78
time (sec)	N/A	0.011	0.121	0.533	0.537	1.172	0.000	0.754	0.052

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	50	58	58	0	54	48
N.S.	1	1.00	0.90	0.81	0.94	0.94	0.00	0.87	0.77
time (sec)	N/A	0.010	0.121	0.473	0.535	1.407	0.000	0.569	0.195

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	56	32	41	60	0	31	30
N.S.	1	1.00	1.30	0.74	0.95	1.40	0.00	0.72	0.70
time (sec)	N/A	0.008	0.091	0.514	0.533	1.017	0.000	0.515	0.150

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	50	58	58	0	54	48
N.S.	1	1.00	1.03	0.85	0.98	0.98	0.00	0.92	0.81
time (sec)	N/A	0.009	0.170	0.551	0.511	2.582	0.000	0.541	0.193

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	46	46	86	0	0	36
N.S.	1	1.00	1.00	0.78	0.78	1.46	0.00	0.00	0.61
time (sec)	N/A	0.009	0.098	0.794	0.536	2.450	0.000	0.000	0.054

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	50	58	58	0	54	48
N.S.	1	1.00	0.90	0.81	0.94	0.94	0.00	0.87	0.77
time (sec)	N/A	0.009	0.126	0.498	0.528	2.651	0.000	0.732	0.221

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	56	32	41	60	0	31	30
N.S.	1	1.00	1.44	0.82	1.05	1.54	0.00	0.79	0.77
time (sec)	N/A	0.006	0.089	0.522	0.527	2.517	0.000	1.617	0.052

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	24	9	8	20	0	40	20
N.S.	1	1.00	1.71	0.64	0.57	1.43	0.00	2.86	1.43
time (sec)	N/A	0.005	0.045	0.509	0.484	2.700	0.000	1.509	0.199

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	25	7	8	33	0	24	6
N.S.	1	1.00	2.50	0.70	0.80	3.30	0.00	2.40	0.60
time (sec)	N/A	0.005	0.051	0.566	0.502	1.984	0.000	1.268	0.132

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	22	20	0	41	20
N.S.	1	1.00	0.96	1.20	0.88	0.80	0.00	1.64	0.80
time (sec)	N/A	0.004	0.046	0.526	0.493	2.219	0.000	1.545	0.218

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	27	15	16	38	0	53	26
N.S.	1	1.00	1.50	0.83	0.89	2.11	0.00	2.94	1.44
time (sec)	N/A	0.007	0.056	0.624	0.537	2.258	0.000	1.727	0.225

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	39	15	16	40	0	36	16
N.S.	1	1.00	2.05	0.79	0.84	2.11	0.00	1.89	0.84
time (sec)	N/A	0.007	0.068	0.526	0.498	1.843	0.000	1.322	0.137

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	30	28	38	0	54	26
N.S.	1	1.00	0.86	0.86	0.80	1.09	0.00	1.54	0.74
time (sec)	N/A	0.005	0.067	0.461	0.521	1.735	0.000	1.160	0.236

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	12	11	40	0	31	11
N.S.	1	1.00	1.76	0.71	0.65	2.35	0.00	1.82	0.65
time (sec)	N/A	0.005	0.056	0.494	0.527	1.268	0.000	0.901	0.170

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	27	30	28	37	0	54	26
N.S.	1	1.00	0.84	0.94	0.88	1.16	0.00	1.69	0.81
time (sec)	N/A	0.005	0.056	0.506	0.510	1.838	0.000	0.676	0.228

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	16	67	0	0	17
N.S.	1	1.00	1.00	0.79	0.48	2.03	0.00	0.00	0.52
time (sec)	N/A	0.005	0.067	0.624	0.546	1.544	0.000	0.000	0.136

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	30	28	38	0	54	26
N.S.	1	1.00	0.86	0.86	0.80	1.09	0.00	1.54	0.74
time (sec)	N/A	0.005	0.066	0.476	0.539	1.701	0.000	0.639	0.226

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	30	12	11	40	0	31	11
N.S.	1	1.00	2.31	0.92	0.85	3.08	0.00	2.38	0.85
time (sec)	N/A	0.003	0.055	0.620	0.495	1.647	0.000	0.660	0.164

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	44	51	16	137	0	60	40
N.S.	1	1.00	2.00	2.32	0.73	6.23	0.00	2.73	1.82
time (sec)	N/A	0.009	0.142	0.676	0.263	2.123	0.000	0.746	0.423

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	123	44	19	141	0	65	46
N.S.	1	1.00	5.35	1.91	0.83	6.13	0.00	2.83	2.00
time (sec)	N/A	0.009	0.223	0.602	0.524	1.461	0.000	1.546	0.406

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	123	44	19	143	0	63	44
N.S.	1	1.00	6.15	2.20	0.95	7.15	0.00	3.15	2.20
time (sec)	N/A	0.008	0.214	0.605	0.481	1.938	0.000	1.653	0.400

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	26	38	0	17	15
N.S.	1	1.00	1.00	0.95	1.37	2.00	0.00	0.89	0.79
time (sec)	N/A	0.002	0.082	0.461	0.289	1.331	0.000	1.380	0.053

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	30	41	0	17	17
N.S.	1	1.00	1.00	0.87	1.30	1.78	0.00	0.74	0.74
time (sec)	N/A	0.002	0.092	0.520	0.289	2.176	0.000	1.389	0.057

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	33	33	30	29	0	29	19
N.S.	1	1.00	1.43	1.43	1.30	1.26	0.00	1.26	0.83
time (sec)	N/A	0.003	0.075	0.502	0.268	1.288	0.000	1.630	0.055

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	59	49	0	36	29
N.S.	1	1.00	1.00	0.93	1.37	1.14	0.00	0.84	0.67
time (sec)	N/A	0.005	0.139	0.487	0.277	2.098	0.000	1.627	0.035

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.083	0.206	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.014	0.162	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.011	0.168	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.013	0.193	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.007	0.010	0.158	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.055	0.131	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	17	16	19	20	16	32
N.S.	1	1.00	0.94	0.94	0.89	1.06	1.11	0.89	1.78
time (sec)	N/A	0.001	0.014	0.399	0.288	1.280	0.007	3.322	0.390

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.011	0.182	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.011	0.189	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.012	0.185	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.043	0.160	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.014	0.217	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [98] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	3	1.00	13	0.231
2	A	6	3	1.00	15	0.200
3	A	5	3	1.00	15	0.200
4	A	4	3	1.00	15	0.200
5	A	3	3	1.00	15	0.200
6	A	6	3	1.00	13	0.231
7	A	5	3	1.00	13	0.231
8	A	4	3	1.00	13	0.231
9	A	3	3	1.00	13	0.231
10	A	3	3	1.00	13	0.231
11	A	3	3	1.00	13	0.231
12	A	4	3	1.00	11	0.273
13	A	3	3	1.00	11	0.273
14	A	3	3	1.00	11	0.273
15	A	3	3	1.00	11	0.273
16	A	3	2	1.00	13	0.154
17	A	2	2	1.00	15	0.133
18	A	1	1	1.00	15	0.067
19	A	2	2	1.00	15	0.133
20	A	3	2	1.00	15	0.133
21	A	2	2	1.00	13	0.154
22	A	1	1	1.00	13	0.077
23	A	2	2	1.00	13	0.154
24	A	3	2	1.00	13	0.154
25	A	2	2	1.00	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	15	0.133
27	A	2	2	1.00	13	0.154
28	A	2	2	1.00	11	0.182
29	A	2	2	1.00	11	0.182
30	A	6	5	1.00	13	0.385
31	A	5	5	1.00	13	0.385
32	A	4	4	1.00	13	0.308
33	A	5	5	1.00	13	0.385
34	A	6	5	1.00	13	0.385
35	A	8	7	1.00	13	0.538
36	A	7	7	1.00	13	0.538
37	A	6	6	1.00	13	0.462
38	A	7	7	1.00	13	0.538
39	A	8	7	1.00	13	0.538
40	A	5	4	1.00	13	0.308
41	A	4	4	1.00	13	0.308
42	A	4	4	1.00	13	0.308
43	A	3	3	1.00	13	0.231
44	A	3	3	1.00	13	0.231
45	A	4	4	1.00	13	0.308
46	A	5	4	1.00	13	0.308
47	A	6	4	1.00	13	0.308
48	A	1	1	1.00	11	0.091
49	A	2	1	1.00	9	0.111
50	A	2	1	1.00	9	0.111
51	A	2	1	1.00	9	0.111
52	A	1	0	1.00	7	0.000
53	A	1	1	1.00	9	0.111
54	A	2	2	1.00	9	0.222
55	A	3	2	1.00	9	0.222
56	A	5	3	1.00	11	0.273
57	A	4	3	1.00	11	0.273
58	A	3	3	1.00	11	0.273
59	A	2	2	1.00	11	0.182
60	A	1	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	11	0.182
62	A	3	2	1.00	11	0.182
63	A	4	2	1.00	11	0.182
64	A	1	1	1.00	14	0.071
65	A	1	1	1.00	14	0.071
66	A	2	2	1.00	14	0.143
67	A	1	1	1.00	14	0.071
68	A	1	1	1.00	14	0.071
69	A	2	2	1.00	14	0.143
70	A	1	1	1.00	14	0.071
71	A	2	2	1.00	14	0.143
72	A	1	1	1.00	14	0.071
73	A	2	2	1.00	14	0.143
74	A	3	2	1.00	23	0.087
75	A	3	2	1.00	23	0.087
76	A	3	2	1.00	23	0.087
77	A	3	2	1.00	23	0.087
78	A	2	2	1.00	12	0.167
79	A	2	2	1.00	15	0.133
80	A	2	2	1.00	12	0.167
81	A	3	2	1.00	12	0.167
82	A	3	2	1.00	12	0.167
83	B	3	2	2.83	10	0.200
84	A	2	2	1.00	12	0.167
85	A	2	2	1.00	12	0.167
86	A	2	2	1.00	12	0.167
87	A	2	2	1.00	12	0.167
88	A	2	2	1.00	12	0.167
89	A	2	2	1.00	13	0.154
90	A	2	2	1.00	14	0.143
91	A	3	3	1.00	12	0.250
92	A	3	3	1.00	12	0.250
93	A	4	3	1.00	12	0.250
94	A	4	3	1.00	12	0.250
95	A	3	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	13	0.231
97	A	3	3	1.00	14	0.214
98	A	2	2	1.00	40	0.050
99	A	3	2	1.76	30	0.067
100	A	3	2	1.87	31	0.065
101	A	2	2	1.00	14	0.143
102	A	2	2	1.00	16	0.125
103	A	3	3	1.00	14	0.214
104	A	3	3	1.00	14	0.214
105	A	3	3	1.00	14	0.214
106	A	3	3	1.00	14	0.214
107	A	3	3	1.00	14	0.214
108	A	3	3	1.00	14	0.214
109	A	3	3	1.00	14	0.214
110	A	3	3	1.00	14	0.214
111	A	3	3	1.00	14	0.214
112	A	3	3	1.00	14	0.214
113	A	3	3	1.00	14	0.214
114	A	2	2	1.00	14	0.143
115	A	2	2	1.00	14	0.143
116	A	2	2	1.00	14	0.143
117	A	2	2	1.00	14	0.143
118	A	2	2	1.00	14	0.143
119	A	2	2	1.00	14	0.143
120	A	2	2	1.00	14	0.143
121	A	2	2	1.00	14	0.143
122	A	2	2	1.00	14	0.143
123	A	2	2	1.00	14	0.143
124	A	2	2	1.00	14	0.143
125	A	2	2	1.00	27	0.074
126	A	2	2	1.00	30	0.067
127	A	2	2	1.00	28	0.071
128	A	1	1	1.00	12	0.083
129	A	1	1	1.00	14	0.071
130	A	1	1	1.00	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	14	0.143
132	A	1	1	1.00	12	0.083
133	A	2	2	1.00	12	0.167
134	A	2	2	1.00	12	0.167
135	A	2	2	1.00	12	0.167
136	A	2	2	1.00	12	0.167
137	A	1	1	1.00	10	0.100
138	A	1	1	1.00	7	0.143
139	A	2	2	1.00	12	0.167
140	A	2	2	1.00	12	0.167
141	A	2	2	1.00	12	0.167
142	A	2	2	1.00	12	0.167
143	A	2	2	1.00	12	0.167

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(bx+cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=147

$$-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3}-\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2}+\frac{(b+2cx)\left(bx+cx^2\right)^{7/}}{16c}$$

```
[Out] 35/6144*b^4*(2*c*x+b)*(c*x^2+b*x)^(3/2)/c^3-7/384*b^2*(2*c*x+b)*(c*x^2+b*x)
^(5/2)/c^2+1/16*(2*c*x+b)*(c*x^2+b*x)^(7/2)/c+35/16384*b^8*arctanh(x*c^(1/2
)/(c*x^2+b*x)^(1/2))/c^(9/2)-35/16384*b^6*(2*c*x+b)*(c*x^2+b*x)^(1/2)/c^4
```

Rubi [A]
time = 0.04, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {626, 634, 212}

$$\frac{35b^8\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}}-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3}-\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2}+\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(7/2),x]
```

```
[Out] (-35*b^6*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(16384*c^4) + (35*b^4*(b + 2*c*x)*(
b*x + c*x^2)^(3/2))/(6144*c^3) - (7*b^2*(b + 2*c*x)*(b*x + c*x^2)^(5/2))/(3
84*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(7/2))/(16*c) + (35*b^8*ArcTanh[(Sqrt[
c]*x)/Sqrt[b*x + c*x^2]])/(16384*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{7/2}\,dx = \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} - \frac{\left(7b^2\right)\int\left(bx+cx^2\right)^{5/2}\,dx}{32c}$$

$$= -\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} + \frac{\left(35b^4\right)\int\left(bx+cx^2\right)^{3/2}\,dx}{768c^2}$$

$$= \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c} -$$

$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)}{384c^2}$$

$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)}{384c^2}$$

$$= -\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)}{384c^2}$$

Mathematica [A]
time = 0.18, size = 157, normalized size = 1.07

$$\frac{\sqrt{x}\,\sqrt{b+cx}\left(\sqrt{c}\,\sqrt{x}\,\sqrt{b+cx}\,(-105b^7+70b^6cx-56b^5c^2x^2+48b^4c^3x^3+10880b^3c^4x^4+25856b^2c^5x^5+21504bc^6x^6+6144c^7x^7)-105b^8\log\left(-\sqrt{c}\,\sqrt{x}+\sqrt{b+cx}\right)\right)}{49152c^{9/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(7/2),x]
[Out] (Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-105*b^7 + 70*b^6*c*
x - 56*b^5*c^2*x^2 + 48*b^4*c^3*x^3 + 10880*b^3*c^4*x^4 + 25856*b^2*c^5*x^5
 + 21504*b*c^6*x^6 + 6144*c^7*x^7) - 105*b^8*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[
b + c*x]]))/(49152*c^(9/2)*Sqrt[x*(b + c*x)])
```

Maple [A]
time = 0.40, size = 149, normalized size = 1.01

method	result
risch	$-\frac{\left(-6144c^7x^7-21504b\,c^6x^6-25856b^2c^5x^5-10880b^3c^4x^4-48b^4c^3x^3+56b^5c^2x^2-70b^6cx+105b^7\right)x(cx+b)}{49152c^4\sqrt{x\,(cx+b)}}+\frac{35b^8\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c}\right.}{32768c^{\frac{9}{2}}}$

default	$\frac{(2cx+b)(cx^2+bx)^{\frac{7}{2}}}{16c}-\frac{7b^2\left(\frac{(2cx+b)(cx^2+bx)^{\frac{5}{2}}}{12c}-\frac{5b^2\left(\frac{(2cx+b)(cx^2+bx)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{cx^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+}\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{24c}\right.}{32c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(2*c*x+b)*(c*x^2+b*x)^(7/2)/c-7/32*b^2/c*(1/12*(2*c*x+b)/c*(c*x^2+b*x)
^(5/2)-5/24*b^2/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x)^(3/2)-3/16*b^2/c*(1/4*(2*c*x
+b)/c*(c*x^2+b*x)^(1/2)-1/8*b^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^
(1/2)))))
```

Maxima [A]
time = 0.28, size = 180, normalized size = 1.22

$$\frac{1}{8}\left(cx^2+bx\right)^{\frac{7}{2}}x-\frac{35\sqrt{cx^2+bx}\,b^6x}{8192\,c^3}+\frac{35\left(cx^2+bx\right)^{\frac{3}{2}}b^4x}{3072\,c^2}-\frac{7\left(cx^2+bx\right)^{\frac{5}{2}}b^2x}{192\,c}+\frac{35\,b^8\log\left(2\,cx+b+2\sqrt{cx^2+bx}\,\sqrt{c}\right)}{32768\,c^{\frac{9}{2}}}-\frac{35\sqrt{cx^2+bx}\,b^7}{16384\,c^4}+\frac{35\left(cx^2+bx\right)^{\frac{3}{2}}b^5}{6144\,c^3}-\frac{7\left(cx^2+bx\right)^{\frac{5}{2}}b^3}{384\,c^2}+\frac{\left(cx^2+bx\right)^{\frac{7}{2}}b}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2 + b*x)^(7/2)*x - 35/8192*sqrt(c*x^2 + b*x)*b^6*x/c^3 + 35/3072*(
c*x^2 + b*x)^(3/2)*b^4*x/c^2 - 7/192*(c*x^2 + b*x)^(5/2)*b^2*x/c + 35/32768
*b^8*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(9/2) - 35/16384*sqrt(c
*x^2 + b*x)*b^7/c^4 + 35/6144*(c*x^2 + b*x)^(3/2)*b^5/c^3 - 7/384*(c*x^2 +
b*x)^(5/2)*b^3/c^2 + 1/16*(c*x^2 + b*x)^(7/2)*b/c
```

Fricas [A]
time = 1.30, size = 258, normalized size = 1.76

$$\left[\frac{105\,b^8\sqrt{c}\,\log\left(2\,cx+b+2\sqrt{cx^2+bx}\,\sqrt{c}\right)+2\left(6144\,c^8x^7+21504\,bc^7x^6+25856\,b^2c^6x^5+10880\,b^3c^5x^4+48\,b^4c^4x^3-56\,b^5c^3x^2+70\,b^6c^2x-105\,b^7c\right)\sqrt{cx^2+bx}}{98304\,c^5},-\frac{105\,b^8\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^2+bx}\,\sqrt{-c}}{cx}\right)-\left(6144\,c^8x^7+21504\,bc^7x^6+25856\,b^2c^6x^5+10880\,b^3c^5x^4+48\,b^4c^4x^3-56\,b^5c^3x^2+70\,b^6c^2x-105\,b^7c\right)\sqrt{cx^2+bx}}{49152\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/98304*(105*b^8*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*
(6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4 + 4
8*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 + b*x
))/c^5, -1/49152*(105*b^8*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x))
 - (6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4
+ 48*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 +
b*x))/c^5]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx + cx^2\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(7/2), x)
```

Giac [**A**]
time = 2.46, size = 132, normalized size = 0.90

$$-\frac{35\,b^8\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx}\right)\sqrt{c}\,-b\right|\right)}{32768\,c^{\frac{9}{2}}}-\frac{1}{49152}\left(\frac{105\,b^7}{c^4}-2\left(\frac{35\,b^6}{c^3}-4\left(\frac{7\,b^5}{c^2}-2\left(\frac{3\,b^4}{c}+8\left(85\,b^3+2\left(101\,b^2c+12\left(2\,c^3x+7\,bc^2\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="giac")
```

```
[Out] -35/32768*b^8*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(9
/2) - 1/49152*(105*b^7/c^4 - 2*(35*b^6/c^3 - 4*(7*b^5/c^2 - 2*(3*b^4/c + 8*
(85*b^3 + 2*(101*b^2*c + 12*(2*c^3*x + 7*b*c^2)*x)*x)*x)*x)*x)*x)*sqrt(c*x^
2 + b*x)
```

Mupad [**B**]
time = 0.74, size = 151, normalized size = 1.03

$$\frac{\left(c\,x^2+b\,x\right)^{7/2}\,\left(\frac{b}{2}+c\,x\right)}{8\,c}-\frac{7\,b^2\,\left(\frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(\frac{b}{2}+c\,x\right)}{6\,c}-\frac{5\,b^2\,\left(\frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(\frac{b}{2}+c\,x\right)}{4\,c}-\frac{3\,b^2\,\left(\sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}\right)}{24\,c}\right)}{32\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(7/2),x)
```

```
[Out] ((b*x + c*x^2)^(7/2)*(b/2 + c*x))/(8*c) - (7*b^2*(((b*x + c*x^2)^(5/2)*(b/2
 + c*x))/(6*c) - (5*b^2*(((b*x + c*x^2)^(3/2)*(b/2 + c*x))/(4*c) - (3*b^2*(
(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x +
 c*x^2)^(1/2)))/(8*c^(3/2))))/(16*c)))/(24*c)))/(32*c)
```

3.2 $\int \left(3ix+4x^2\right)^{7/2}\,dx$

Optimal. Leaf size=121

$$\frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304}+\frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072}+\frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048}+\frac{1}{64}(3i+8x)\left(3ix+\right.$$

[Out] 945/131072*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+21/2048*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+1/64*(3*I+8*x)*(3*I*x+4*x^2)^(7/2)-229635/16777216*I*arcsin(-1+8/3*I*x)+25515/4194304*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {626, 633, 221}

$$\frac{229635i\text{ArcSin}\left(1-\frac{8ix}{3}\right)}{16777216}+\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}+\frac{21(8x+3i)\left(4x^2+3ix\right)^{5/2}}{2048}+\frac{945(8x+3i)\left(4x^2+3ix\right)^{3/2}}{131072}+\frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (25515*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/4194304 + (945*(3*I + 8*x)*((3*I)
*x + 4*x^2)^(3/2))/131072 + (21*(3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/2048 +
 ((3*I + 8*x)*((3*I)*x + 4*x^2)^(7/2))/64 + ((229635*I)/16777216)*ArcSin[1
- ((8*I)/3)*x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{7/2}\,dx = \frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2}+\frac{63}{128}\int \left(3ix+4x^2\right)^{5/2}\,dx$$

$$= \frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048}+\frac{1}{64}(3i+8x)\left(3ix+4x^2\right)^{7/2}+\frac{945\int \left(3ix+4x^2\right)^{3/2}\,dx}{4096}$$

$$= \frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072}+\frac{21(3i+8x)\left(3ix+4x^2\right)^{5/2}}{2048}+\frac{1}{64}(3i+8x)\left(3ix+4x^2\right)$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304}+\frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072}+\frac{21(3i+8x)\left(3ix+4x^2\right.}{2048}$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304}+\frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072}+\frac{21(3i+8x)\left(3ix+4x^2\right.}{2048}$$

$$= \frac{25515(3i+8x)\sqrt{3ix+4x^2}}{4194304}+\frac{945(3i+8x)\left(3ix+4x^2\right)^{3/2}}{131072}+\frac{21(3i+8x)\left(3ix+4x^2\right.}{2048}$$

Mathematica [A]
time = 0.12, size = 117, normalized size = 0.97

$$\frac{\sqrt{x}\sqrt{3i+4x}\left(2\sqrt{x}\sqrt{3i+4x}\,(76545i-68040x-72576ix^2+82944x^3-25067520ix^4-79429632x^5+88080384ix^6+33554432x^7)-229635\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)\right)}{8388608\sqrt{x(3i+4x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[3*I + 4*x]*(2*Sqrt[x]*Sqrt[3*I + 4*x]*(76545*I - 68040*x - (7
2576*I)*x^2 + 82944*x^3 - (25067520*I)*x^4 - 79429632*x^5 + (88080384*I)*x^
6 + 33554432*x^7) - 229635*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]]))/(8388608*Sqr
t[x*(3*I + 4*x)])
```

Maple [A]
time = 0.56, size = 91, normalized size = 0.75

method	result
risch	$\frac{(33554432x^7+88080384ix^6-79429632x^5-25067520ix^4+82944x^3-72576ix^2-68040x+76545i)x(3i+4x)}{4194304\sqrt{x(3i+4x)}}+\frac{229635\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{16777216}$
default	$\frac{(3i+8x)(4x^2+3ix)^{\frac{7}{2}}}{64}+\frac{21(3i+8x)(4x^2+3ix)^{\frac{5}{2}}}{2048}+\frac{945(3i+8x)(4x^2+3ix)^{\frac{3}{2}}}{131072}+\frac{25515(3i+8x)\sqrt{4x^2+3ix}}{4194304}+\frac{229635\,\mathrm{arcsinh}}{1677721}$
trager	$\left(21ix^6+8x^7-\frac{765}{128}ix^4-\frac{303}{16}x^5-\frac{567}{32768}ix^2+\frac{81}{4096}x^3+\frac{76545}{4194304}i-\frac{8505}{524288}x\right)\sqrt{4x^2+3ix}+\frac{229635\ln\left(44\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(3*I+8*x)*(3*I*x+4*x^2)^(7/2)+21/2048*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+94
5/131072*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+25515/4194304*(3*I+8*x)*(3*I*x+4*x^2
)^(1/2)+229635/16777216*arcsinh(I+8/3*x)
```

Maxima [A]
time = 0.54, size = 130, normalized size = 1.07

$$\frac{1}{8}\left(4x^2+3ix\right)^{\frac{7}{2}}x+\frac{3}{64}i\left(4x^2+3ix\right)^{\frac{7}{2}}+\frac{21}{256}\left(4x^2+3ix\right)^{\frac{5}{2}}x+\frac{63}{2048}i\left(4x^2+3ix\right)^{\frac{5}{2}}+\frac{945}{16384}\left(4x^2+3ix\right)^{\frac{3}{2}}x+\frac{2835}{131072}i\left(4x^2+3ix\right)^{\frac{3}{2}}+\frac{25515}{524288}\sqrt{4x^2+3ix}\,x+\frac{76545}{4194304}i\sqrt{4x^2+3ix}+\frac{229635}{16777216}\log\left(8x+4\sqrt{4x^2+3ix}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x^2 + 3*I*x)^(7/2)*x + 3/64*I*(4*x^2 + 3*I*x)^(7/2) + 21/256*(4*x^2
+ 3*I*x)^(5/2)*x + 63/2048*I*(4*x^2 + 3*I*x)^(5/2) + 945/16384*(4*x^2 + 3*I
*x)^(3/2)*x + 2835/131072*I*(4*x^2 + 3*I*x)^(3/2) + 25515/524288*sqrt(4*x^2
 + 3*I*x)*x + 76545/4194304*I*sqrt(4*x^2 + 3*I*x) + 229635/16777216*log(8*x
 + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [A]
time = 1.72, size = 69, normalized size = 0.57

$$\frac{1}{4194304}\left(33554432x^7+88080384ix^6-79429632x^5-25067520ix^4+82944x^3-72576ix^2-68040x+76545i\right)\sqrt{4x^2+3ix}-\frac{229635}{16777216}\log\left(-2x+\sqrt{4x^2+3ix}-\frac{3}{4}i\right)-\frac{1165671}{268435456}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4194304*(33554432*x^7 + 88080384*I*x^6 - 79429632*x^5 - 25067520*I*x^4 +
82944*x^3 - 72576*I*x^2 - 68040*x + 76545*I)*sqrt(4*x^2 + 3*I*x) - 229635/1
6777216*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 1165671/268435456
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(4x^2+3ix\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(7/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(7/2), x)
```

Giac [A]
time = 1.26, size = 140, normalized size = 1.16

$$\frac{1}{8388608}\left(8\left(16\left(8\left(32\left(8\left(16\left(8x+21i\right)x-303\right)x-765i\right)x+81\right)x-567i\right)x-8505\right)x+76545i\right)\sqrt{8x^2+2\sqrt{16x^2+9}\,x}\left(\frac{3ix}{4x^2+\sqrt{16x^4+9x^2}}+1\right)-\frac{229635}{16777216}\log\left(2\sqrt{8x^2+2\sqrt{16x^2+9}\,x}\left(\frac{3ix}{4x^2+\sqrt{16x^4+9x^2}}+1\right)-8x-3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/8388608*(8*(16*(8*(32*(8*(16*(8*x + 21*I)*x - 303)*x - 765*I)*x + 81)*x -
 567*I)*x - 8505)*x + 76545*I)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4
*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 229635/16777216*log(2*sqrt(8*x^2 + 2*sq
rt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

Mupad [B]
time = 0.29, size = 100, normalized size = 0.83

$$\frac{229635\ln\left(x+\frac{\sqrt{x\,(4x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{16777216}+\frac{945\left(4x+\frac{3}{2}\mathrm{i}\right)\left(4x^2+x\,3\mathrm{i}\right)^{3/2}}{65536}+\frac{21\left(4x+\frac{3}{2}\mathrm{i}\right)\left(4x^2+x\,3\mathrm{i}\right)^{5/2}}{1024}+\frac{\left(4x+\frac{3}{2}\mathrm{i}\right)\left(4x^2+x\,3\mathrm{i}\right)^{7/2}}{32}+\frac{25515\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\sqrt{4x^2+x\,3\mathrm{i}}}{262144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*3i + 4*x^2)^(7/2),x)
```

```
[Out] (229635*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/16777216 + (945*(4*x + 3i/2
)*(x*3i + 4*x^2)^(3/2))/65536 + (21*(4*x + 3i/2)*(x*3i + 4*x^2)^(5/2))/1024
 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(7/2))/32 + (25515*(x/2 + 3i/16)*(x*3i + 4*
x^2)^(1/2))/262144
```

3.3 $\int \left(3ix+4x^2\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768}+\frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024}+\frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2}+\frac{3645i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{131072}$$

```
[Out] 15/1024*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+1/48*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)-36
45/131072*I*arcsin(-1+8/3*I*x)+405/32768*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {626, 633, 221}

$$\frac{3645i\text{ArcSin}\left(1-\frac{8ix}{3}\right)}{131072}+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}+\frac{15(8x+3i)\left(4x^2+3ix\right)^{3/2}}{1024}+\frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (405*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/32768 + (15*(3*I + 8*x)*((3*I)*x +
4*x^2)^(3/2))/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/48 + ((3645*I)/1
31072)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{5/2} dx = \frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2}+\frac{15}{32}\int \left(3ix+4x^2\right)^{3/2} dx$$

$$= \frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024}+\frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2}+\frac{405\int \sqrt{3ix+4x^2}\,dx}{2048}$$

$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768}+\frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024}+\frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2}$$

$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768}+\frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024}+\frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2}$$

$$= \frac{405(3i+8x)\sqrt{3ix+4x^2}}{32768}+\frac{15(3i+8x)\left(3ix+4x^2\right)^{3/2}}{1024}+\frac{1}{48}(3i+8x)\left(3ix+4x^2\right)^{5/2}$$

Mathematica [A]
time = 0.09, size = 86, normalized size = 0.91

$$\frac{\sqrt{x(3i+4x)}\left(7290i-6480x-6912ix^2-497664x^3+983040ix^4+524288x^5-\frac{10935\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{\sqrt{x}\sqrt{3i+4x}}\right)}{196608}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(7290*I - 6480*x - (6912*I)*x^2 - 497664*x^3 + (983040
*I)*x^4 + 524288*x^5 - (10935*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(Sqrt[x]*S
qrt[3*I + 4*x])))/196608
```

Maple [A]
time = 0.41, size = 71, normalized size = 0.75

method	result
risch	$\frac{(262144x^5+491520ix^4-248832x^3-3456ix^2-3240x+3645i)x(3i+4x)}{98304\sqrt{x(3i+4x)}}+\frac{3645\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{131072}$
default	$\frac{(3i+8x)(4x^2+3ix)^{\frac{5}{2}}}{48}+\frac{15(3i+8x)(4x^2+3ix)^{\frac{3}{2}}}{1024}+\frac{405(3i+8x)\sqrt{4x^2+3ix}}{32768}+\frac{3645\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{131072}$
trager	$\left(5ix^4+\frac{8}{3}x^5-\frac{9}{256}ix^2-\frac{81}{32}x^3+\frac{1215}{32768}i-\frac{135}{4096}x\right)\sqrt{4x^2+3ix}+\frac{3645\ln\left(440x+144+165i-192i\sqrt{4x^2+3ix}\right.}{131072}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+15/1024*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+40
5/32768*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+3645/131072*arcsinh(I+8/3*x)
```

Maxima [A]
time = 0.53, size = 103, normalized size = 1.08

$$\frac{1}{6}\left(4x^2+3ix\right)^{\frac{5}{2}}x+\frac{1}{16}i\left(4x^2+3ix\right)^{\frac{5}{2}}+\frac{15}{128}\left(4x^2+3ix\right)^{\frac{3}{2}}x+\frac{45}{1024}i\left(4x^2+3ix\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{4x^2+3ix}\,x+\frac{1215}{32768}i\sqrt{4x^2+3ix}+\frac{3645}{131072}\log\left(8x+4\sqrt{4x^2+3ix}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(4*x^2 + 3*I*x)^(5/2)*x + 1/16*I*(4*x^2 + 3*I*x)^(5/2) + 15/128*(4*x^2
+ 3*I*x)^(3/2)*x + 45/1024*I*(4*x^2 + 3*I*x)^(3/2) + 405/4096*sqrt(4*x^2 +
3*I*x)*x + 1215/32768*I*sqrt(4*x^2 + 3*I*x) + 3645/131072*log(8*x + 4*sqrt(
4*x^2 + 3*I*x) + 3*I)
```

Fricas [A]
time = 1.31, size = 59, normalized size = 0.62

$$\frac{1}{98304}\left(262144\,x^5+491520i\,x^4-248832\,x^3-3456i\,x^2-3240\,x+3645i\right)\sqrt{4x^2+3ix}-\frac{3645}{131072}\log\left(-2x+\sqrt{4x^2+3ix}-\frac{3}{4}i\right)-\frac{8991}{1048576}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(262144*x^5 + 491520*I*x^4 - 248832*x^3 - 3456*I*x^2 - 3240*x + 364
5*I)*sqrt(4*x^2 + 3*I*x) - 3645/131072*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4
*I) - 8991/1048576
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(4x^2+3ix\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(5/2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(63) = 126.
time = 2.63, size = 130, normalized size = 1.37

$$\frac{1}{196608}\left(8\left(16\left(8\left(32\left(8x+15i\right)x-243\right)x-27i\right)x-405\right)x+3645i\right)\sqrt{8x^2+2\sqrt{16x^2+9}\,x\left(\frac{3ix}{4x^2+\sqrt{16x^4+9x^2}}+1\right)}-\frac{3645}{131072}\log\left(2\sqrt{8x^2+2\sqrt{16x^2+9}\,x\left(\frac{3ix}{4x^2+\sqrt{16x^4+9x^2}}+1\right)}-8x-3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/196608*(8*(16*(8*(32*(8*x + 15*I)*x - 243)*x - 27*I)*x - 405)*x + 3645*I)
*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) +
 1) - 3645/131072*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 +
sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

Mupad [B]
time = 0.30, size = 80, normalized size = 0.84

$$\frac{3645\,\ln\left(x+\frac{\sqrt{x\,\left(4\,x+3\mathrm{i}\right)}}{2}+\frac{3}{8}\mathrm{i}\right)}{131072}+\frac{15\,\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{3/2}}{512}+\frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{5/2}}{24}+\frac{405\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*3i + 4*x^2)^(5/2),x)
```

```
[Out] (3645*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/131072 + (15*(4*x + 3i/2)*(x*
3i + 4*x^2)^(3/2))/512 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(5/2))/24 + (405*(x/2
 + 3i/16)*(x*3i + 4*x^2)^(1/2))/2048
```

3.4 $\int (3ix+4x^2)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

```
[Out] 1/32*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)-243/4096*I*arcsin(-1+8/3*I*x)+27/1024*(3
*I+8*x)*(3*I*x+4*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {626, 633, 221}

$$\frac{243i\text{ArcSin}\left(1-\frac{8ix}{3}\right)}{4096}+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}+\frac{27(8x+3i)\sqrt{4x^2+3ix}}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(3/2),x]
```

```
[Out] (27*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2
)^(3/2))/32 + ((243*I)/4096)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3ix+4x^2\right)^{3/2}\,dx = \frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{27}{64}\int \sqrt{3ix+4x^2}\,dx$$

$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{243\int \frac{1}{\sqrt{3ix+4x^2}}\,dx}{2048}$$

$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{81\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}}\,dx, \right.}{4096}$$

$$= \frac{27(3i+8x)\sqrt{3ix+4x^2}}{1024}+\frac{1}{32}(3i+8x)\left(3ix+4x^2\right)^{3/2}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

Mathematica [A]
time = 0.08, size = 83, normalized size = 1.20

$$\frac{2x(-243+108ix-3744x^2+7680ix^3+4096x^4)-243\sqrt{x}\sqrt{3i+4x}\,\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{2048\sqrt{x(3i+4x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(3/2),x]
[Out] (2*x*(-243 + (108*I)*x - 3744*x^2 + (7680*I)*x^3 + 4096*x^4) - 243*Sqrt[x]*
Sqrt[3*I + 4*x]*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(2048*Sqrt[x*(3*I + 4*x)
])
```

Maple [A]
time = 0.40, size = 51, normalized size = 0.74

method	result
risch	$\frac{(1024x^3+1152ix^2-72x+81i)x(3i+4x)}{1024\sqrt{x(3i+4x)}}+\frac{243\,\text{arcsinh}\left(i+\frac{8x}{3}\right)}{4096}$
default	$\frac{(3i+8x)(4x^2+3ix)^{\frac{3}{2}}}{32}+\frac{27(3i+8x)\sqrt{4x^2+3ix}}{1024}+\frac{243\,\text{arcsinh}\left(i+\frac{8x}{3}\right)}{4096}$
trager	$\left(\frac{9}{8}ix^2+x^3+\frac{81}{1024}i-\frac{9}{128}x\right)\sqrt{4x^2+3ix}+\frac{243\ln\left(440x+144+165i-192i\sqrt{4x^2+3ix}-384ix+220\sqrt{4x^2+3i\right.}{4096}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/32*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+27/1024*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+24
3/4096*arcsinh(I+8/3*x)
```

Maxima [A]
time = 0.54, size = 76, normalized size = 1.10

$$\frac{1}{4}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x+\frac{3}{32}i\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{4\,x^2+3i\,x}\,x+\frac{81}{1024}i\sqrt{4\,x^2+3i\,x}+\frac{243}{4096}\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2 + 3*I*x)^(3/2)*x + 3/32*I*(4*x^2 + 3*I*x)^(3/2) + 27/128*sqrt(4*
x^2 + 3*I*x)*x + 81/1024*I*sqrt(4*x^2 + 3*I*x) + 243/4096*log(8*x + 4*sqrt(
4*x^2 + 3*I*x) + 3*I)
```

Fricas [A]
time = 1.57, size = 49, normalized size = 0.71

$$\frac{1}{1024}\left(1024\,x^3+1152i\,x^2-72\,x+81i\right)\sqrt{4\,x^2+3i\,x}-\frac{243}{4096}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{567}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1024*(1024*x^3 + 1152*I*x^2 - 72*x + 81*I)*sqrt(4*x^2 + 3*I*x) - 243/4096
*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 567/32768
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(4x^2+3ix\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(3/2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(45) = 90$.
time = 1.94, size = 120, normalized size = 1.74

$$\frac{1}{2048}\left(8\left(16\left(8\,x+9i\right)x-9\right)x+81i\right)\sqrt{8\,x^2+2\sqrt{16\,x^2+9}\,x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-\frac{243}{4096}\log\left(2\sqrt{8\,x^2+2\sqrt{16\,x^2+9}\,x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-8\,x-3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2048*(8*(16*(8*x + 9*I)*x - 9)*x + 81*I)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*
x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 243/4096*log(2*sqrt(8*x^2 +
```

```
 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3
*I)
```

Mupad [B]
time = 0.16, size = 60, normalized size = 0.87

$$\frac{243\,\ln\left(x+\frac{\sqrt{x\,\left(4\,x+3\mathrm{i}\right)}}{2}+\frac{3}{8}\mathrm{i}\right)}{4096}+\frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{3/2}}{16}+\frac{27\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*3i + 4*x^2)^(3/2),x)
```

```
[Out] (243*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/4096 + ((4*x + 3i/2)*(x*3i + 4
*x^2)^(3/2))/16 + (27*(x/2 + 3i/16)*(x*3i + 4*x^2)^(1/2))/64
```

3.5 $\int \sqrt{3ix+4x^2}\,dx$

Optimal. Leaf size=43

$$\frac{1}{16}(3i+8x)\sqrt{3ix+4x^2}+\frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

```
[Out] -9/64*I*arcsin(-1+8/3*I*x)+1/16*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {626, 633, 221}

$$\frac{9}{64}i\text{ArcSin}\left(1-\frac{8ix}{3}\right)+\frac{1}{16}\sqrt{4x^2+3ix}\,(8x+3i)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] ((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + ((9*I)/64)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{3ix+4x^2}\,dx = \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2} + \frac{9}{32}\int \frac{1}{\sqrt{3ix+4x^2}}\,dx$$
$$= \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2} + \frac{3}{64}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}}\,dx, x, 3i+8x\right)$$
$$= \frac{1}{16}(3i+8x)\sqrt{3ix+4x^2} + \frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Mathematica [A]
time = 0.05, size = 62, normalized size = 1.44

$$\frac{1}{32}\sqrt{x(3i+4x)}\left(6i+16x-\frac{9\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{\sqrt{x}\sqrt{3i+4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(3*I)*x + 4*x^2],x]
[Out] (Sqrt[x*(3*I + 4*x)]*(6*I + 16*x - (9*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(S
qrt[x]*Sqrt[3*I + 4*x])))/32
```

Maple [A]
time = 0.46, size = 31, normalized size = 0.72

method	result	size
default	$\frac{(3i+8x)\sqrt{4x^2+3ix}}{16}+\frac{9\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{64}$	31
risch	$\frac{(3i+8x)x(3i+4x)}{16\sqrt{x(3i+4x)}}+\frac{9\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{64}$	36
trager	$\left(\frac{3i}{16}+\frac{x}{2}\right)\sqrt{4x^2+3ix}+\frac{9\ln\left(440x+144+165i-192i\sqrt{4x^2+3ix}-384ix+220\sqrt{4x^2+3ix}\right)}{64}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*I*x+4*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+9/64*arcsinh(I+8/3*x)
```

Maxima [A]
time = 0.52, size = 49, normalized size = 1.14

$$\frac{1}{2}\sqrt{4x^2+3ix}\,x+\frac{3}{16}i\sqrt{4x^2+3ix}+\frac{9}{64}\log\left(8x+4\sqrt{4x^2+3ix}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(4*x^2 + 3*I*x)*x + 3/16*I*sqrt(4*x^2 + 3*I*x) + 9/64*log(8*x + 4*s
qrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [A]
time = 1.44, size = 39, normalized size = 0.91

$$\frac{1}{16}\sqrt{4x^2+3ix}\,(8x+3i)-\frac{9}{64}\log\left(-2x+\sqrt{4x^2+3ix}-\frac{3}{4}i\right)-\frac{9}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(4*x^2 + 3*I*x)*(8*x + 3*I) - 9/64*log(-2*x + sqrt(4*x^2 + 3*I*x)
- 3/4*I) - 9/256
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{4x^2+3ix}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(4*x**2 + 3*I*x), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(27) = 54.
time = 2.61, size = 110, normalized size = 2.56

$$\frac{1}{32}\sqrt{8x^2+2\sqrt{16x^2+9}\,x}\,(8x+3i)\left(\frac{3ix}{4x^2+\sqrt{16x^4+9x^2}}+1\right)-\frac{9}{64}\log\left(2\sqrt{8x^2+2\sqrt{16x^2+9}\,x}\left(\frac{3ix}{4x^2+\sqrt{16x^4+9x^2}}+1\right)-8x-3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(16
*x^4 + 9*x^2)) + 1) - 9/64*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/
(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

Mupad [B]
time = 0.09, size = 39, normalized size = 0.91

$$\frac{9\ln\left(x+\frac{\sqrt{x\,(4x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{64}+\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\sqrt{4x^2+x\,3\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*3i + 4*x^2)^(1/2),x)
```

```
[Out] (9*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/64 + (x/2 + 3i/16)*(x*3i + 4*x^2
)^(1/2)
```

3.6 $\int \left(3x-4x^2\right)^{7/2}\,dx$

Optimal. Leaf size=101

$$-\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3-8x)\left(3x-4x^2\right)$$

```
[Out] -945/131072*(3-8*x)*(-4*x^2+3*x)^(3/2)-21/2048*(3-8*x)*(-4*x^2+3*x)^(5/2)-1
/64*(3-8*x)*(-4*x^2+3*x)^(7/2)+229635/16777216*arcsin(-1+8/3*x)-25515/41943
04*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {626, 633, 222}

$$-\frac{229635\text{ArcSin}\left(1-\frac{8x}{3}\right)}{16777216}-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(7/2),x]
```

```
[Out] (-25515*(3 - 8*x)*Sqrt[3*x - 4*x^2])/4194304 - (945*(3 - 8*x)*(3*x - 4*x^2)
^(3/2))/131072 - (21*(3 - 8*x)*(3*x - 4*x^2)^(5/2))/2048 - ((3 - 8*x)*(3*x
- 4*x^2)^(7/2))/64 - (229635*ArcSin[1 - (8*x)/3])/16777216
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3x-4x^2\right)^{7/2}\,dx = -\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}+\frac{63}{128}\int \left(3x-4x^2\right)^{5/2}\,dx$$

$$= -\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}+\frac{945\int \left(3x-4x^2\right)^{3/2}\,dx}{4096}$$

$$= -\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}$$

$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}$$

$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}$$

$$= -\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}$$

Mathematica [A]
time = 0.10, size = 102, normalized size = 1.01

$$\frac{\sqrt{-x(-3+4x)}\left(-2\sqrt{x}\sqrt{-3+4x}\left(76545+68040x+72576x^2+82944x^3-25067520x^4+79429632x^5-88080384x^6+33554432x^7\right)+229635\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)\right)}{8388608\sqrt{x}\sqrt{-3+4x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(7/2),x]
[Out] (Sqrt[-(x*(-3 + 4*x))]*(-2*Sqrt[x]*Sqrt[-3 + 4*x]*(76545 + 68040*x + 72576*
x^2 + 82944*x^3 - 25067520*x^4 + 79429632*x^5 - 88080384*x^6 + 33554432*x^7
) + 229635*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]]))/(8388608*Sqrt[x]*Sqrt[-3 + 4*
x])
```

Maple [A]
time = 0.42, size = 82, normalized size = 0.81

method	result
risch	$\frac{\left(33554432x^7-88080384x^6+79429632x^5-25067520x^4+82944x^3+72576x^2+68040x+76545\right)x(-3+4x)}{4194304\sqrt{-x\left(-3+4x\right)}}+\frac{229635\arcsin\left(-1+\frac{8x}{3}\right)}{16777216}$
meijerg	$-\frac{688905i\left(-\frac{i\sqrt{\pi}\sqrt{x}\sqrt{3}\left(\frac{33554432}{243}x^7-\frac{29360128}{81}x^6+\frac{26476544}{81}x^5-\frac{2785280}{27}x^4+\frac{1024}{3}x^3+\frac{896}{3}x^2+280x+315\right)\sqrt{-\frac{4x}{3}+1}}{1451520}+\frac{i\sqrt{\pi}\dots}{\dots}\right.}{8192\sqrt{\pi}}$

default	$-\frac{945(3-8x)(-4x^2+3x)^{\frac{3}{2}}}{131072}-\frac{21(3-8x)(-4x^2+3x)^{\frac{5}{2}}}{2048}-\frac{(3-8x)(-4x^2+3x)^{\frac{7}{2}}}{64}+\frac{229635\arcsin(-1+\frac{8x}{3})}{16777216}-\frac{25515(3-8x)\sqrt{-}}{41943}$
trager	$\left(-8x^7+21x^6-\frac{303}{16}x^5+\frac{765}{128}x^4-\frac{81}{4096}x^3-\frac{567}{32768}x^2-\frac{8505}{524288}x-\frac{76545}{4194304}\right)\sqrt{-4x^2+3x}+\frac{229635\,\mathrm{Roo}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -945/131072*(3-8*x)*(-4*x^2+3*x)^(3/2)-21/2048*(3-8*x)*(-4*x^2+3*x)^(5/2)-1
/64*(3-8*x)*(-4*x^2+3*x)^(7/2)+229635/16777216*arcsin(-1+8/3*x)-25515/41943
04*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A]
time = 0.48, size = 117, normalized size = 1.16

$$\frac{1}{8}\left(-4x^2+3x\right)^{\frac{7}{2}}x-\frac{3}{64}\left(-4x^2+3x\right)^{\frac{7}{2}}+\frac{21}{256}\left(-4x^2+3x\right)^{\frac{5}{2}}x-\frac{63}{2048}\left(-4x^2+3x\right)^{\frac{5}{2}}+\frac{945}{16384}\left(-4x^2+3x\right)^{\frac{3}{2}}x-\frac{2835}{131072}\left(-4x^2+3x\right)^{\frac{3}{2}}+\frac{25515}{524288}\sqrt{-4x^2+3x}\,x-\frac{76545}{4194304}\sqrt{-4x^2+3x}-\frac{229635}{16777216}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*x^2 + 3*x)^(7/2)*x - 3/64*(-4*x^2 + 3*x)^(7/2) + 21/256*(-4*x^2 + 3
*x)^(5/2)*x - 63/2048*(-4*x^2 + 3*x)^(5/2) + 945/16384*(-4*x^2 + 3*x)^(3/2)
*x - 2835/131072*(-4*x^2 + 3*x)^(3/2) + 25515/524288*sqrt(-4*x^2 + 3*x)*x -
 76545/4194304*sqrt(-4*x^2 + 3*x) - 229635/16777216*arcsin(-8/3*x + 1)
```

Fricas [A]
time = 1.51, size = 68, normalized size = 0.67

$$-\frac{1}{4194304}\left(33554432\,x^7-88080384\,x^6+79429632\,x^5-25067520\,x^4+82944\,x^3+72576\,x^2+68040\,x+76545\right)\sqrt{-4\,x^2+3\,x}-\frac{229635}{8388608}\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/4194304*(33554432*x^7 - 88080384*x^6 + 79429632*x^5 - 25067520*x^4 + 829
44*x^3 + 72576*x^2 + 68040*x + 76545)*sqrt(-4*x^2 + 3*x) - 229635/8388608*a
rctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-4x^2+3x\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(7/2), x)
```

Giac [A]
time = 1.38, size = 57, normalized size = 0.56

$$-\frac{1}{4194304}\left(8\left(16\left(8\left(32\left(8\left(16\left(8x-21\right)x+303\right)x-765\right)x+81\right)x+567\right)x+8505\right)x+76545\right)\sqrt{-4x^2+3x}+\frac{229635}{16777216}\arcsin\left(\frac{8}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4194304*(8*(16*(8*(32*(8*(16*(8*x - 21)*x + 303)*x - 765)*x + 81)*x + 56
7)*x + 8505)*x + 76545)*sqrt(-4*x^2 + 3*x) + 229635/16777216*arcsin(8/3*x -
 1)
```

Mupad [B]
time = 0.17, size = 81, normalized size = 0.80

$$\frac{229635\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{16777216}+\frac{945\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{3/2}}{65536}+\frac{21\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{5/2}}{1024}+\frac{\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{7/2}}{32}+\frac{25515\left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{3x-4x^2}}{262144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2)^(7/2),x)
```

```
[Out] (229635*asin((8*x)/3 - 1))/16777216 + (945*(4*x - 3/2)*(3*x - 4*x^2)^(3/2))
/65536 + (21*(4*x - 3/2)*(3*x - 4*x^2)^(5/2))/1024 + ((4*x - 3/2)*(3*x - 4*
x^2)^(7/2))/32 + (25515*(x/2 - 3/16)*(3*x - 4*x^2)^(1/2))/262144
```

3.7 $\int \left(3x-4x^2\right)^{5/2}\,dx$

Optimal. Leaf size=79

$$-\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{3645\sin^{-1}\left(1-\frac{8x}{3}\right)}{131072}$$

```
[Out] -15/1024*(3-8*x)*(-4*x^2+3*x)^(3/2)-1/48*(3-8*x)*(-4*x^2+3*x)^(5/2)+3645/13
1072*arcsin(-1+8/3*x)-405/32768*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {626, 633, 222}

$$-\frac{3645\text{ArcSin}\left(1-\frac{8x}{3}\right)}{131072}-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (-405*(3 - 8*x)*Sqrt[3*x - 4*x^2])/32768 - (15*(3 - 8*x)*(3*x - 4*x^2)^(3/2
))/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(5/2))/48 - (3645*ArcSin[1 - (8*x)/3])/1
31072
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(3x-4x^2\right)^{5/2} dx = -\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}+\frac{15}{32}\int \left(3x-4x^2\right)^{3/2} dx$$
$$= -\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}+\frac{405\int \sqrt{3x-4x^2}\, dx}{2048}$$
$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}+$$

$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-$$
$$= -\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-$$

Mathematica [A]
time = 0.08, size = 92, normalized size = 1.16

$$\frac{\sqrt{-x(-3+4x)}\left(2\sqrt{x}\sqrt{-3+4x}\left(-3645-3240x-3456x^2+248832x^3-491520x^4+262144x^5\right)+10935\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)\right)}{196608\sqrt{x}\sqrt{-3+4x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[-(x*(-3 + 4*x))]*(2*Sqrt[x]*Sqrt[-3 + 4*x]*(-3645 - 3240*x - 3456*x^2
 + 248832*x^3 - 491520*x^4 + 262144*x^5) + 10935*Log[-2*Sqrt[x] + Sqrt[-3 +
 4*x]]))/(196608*Sqrt[x]*Sqrt[-3 + 4*x])
```

Maple [A]
time = 0.41, size = 64, normalized size = 0.81

method	result
risch	$-\frac{\left(262144x^5-491520x^4+248832x^3-3456x^2-3240x-3645\right)x(-3+4x)}{98304\sqrt{-x\left(-3+4x\right)}}+\frac{3645\arcsin\left(-1+\frac{8x}{3}\right)}{131072}$
default	$-\frac{15(3-8x)\left(-4x^2+3x\right)^{\frac{3}{2}}}{1024}-\frac{(3-8x)\left(-4x^2+3x\right)^{\frac{5}{2}}}{48}+\frac{3645\arcsin\left(-1+\frac{8x}{3}\right)}{131072}-\frac{405(3-8x)\sqrt{-4x^2+3x}}{32768}$
meijerg	$-\frac{10935i\left(-\frac{i\sqrt{\pi}\sqrt{x}\sqrt{3}\left(-\frac{1835008}{243}x^5+\frac{1146880}{81}x^4-7168x^3+\frac{896}{9}x^2+\frac{280}{3}x+105\right)\sqrt{-\frac{4x}{3}+1}}{30240}+\frac{i\sqrt{\pi}\arcsin\left(\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{192}\right.}{1024\sqrt{\pi}}$

trager	$\left(\frac{8}{3}x^5-5x^4+\frac{81}{32}x^3-\frac{9}{256}x^2-\frac{135}{4096}x-\frac{1215}{32768}\right)\sqrt{-4x^2+3x}-\frac{3645\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(8x\,\mathrm{RootOf}\left(\_Z^2+\right.\right.}{131}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -15/1024*(3-8*x)*(-4*x^2+3*x)^(3/2)-1/48*(3-8*x)*(-4*x^2+3*x)^(5/2)+3645/13
1072*arcsin(-1+8/3*x)-405/32768*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A]
time = 0.48, size = 90, normalized size = 1.14

$$\frac{1}{6}\left(-4x^2+3x\right)^{\frac{5}{2}}x-\frac{1}{16}\left(-4x^2+3x\right)^{\frac{5}{2}}+\frac{15}{128}\left(-4x^2+3x\right)^{\frac{3}{2}}x-\frac{45}{1024}\left(-4x^2+3x\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{-4x^2+3x}\,x-\frac{1215}{32768}\sqrt{-4x^2+3x}-\frac{3645}{131072}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-4*x^2 + 3*x)^(5/2)*x - 1/16*(-4*x^2 + 3*x)^(5/2) + 15/128*(-4*x^2 + 3
*x)^(3/2)*x - 45/1024*(-4*x^2 + 3*x)^(3/2) + 405/4096*sqrt(-4*x^2 + 3*x)*x
- 1215/32768*sqrt(-4*x^2 + 3*x) - 3645/131072*arcsin(-8/3*x + 1)
```

Fricas [A]
time = 1.38, size = 58, normalized size = 0.73

$$\frac{1}{98304}\left(262144\,x^5-491520\,x^4+248832\,x^3-3456\,x^2-3240\,x-3645\right)\sqrt{-4\,x^2+3\,x}-\frac{3645}{65536}\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(262144*x^5 - 491520*x^4 + 248832*x^3 - 3456*x^2 - 3240*x - 3645)*s
qrt(-4*x^2 + 3*x) - 3645/65536*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-4x^2+3x\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(5/2), x)
```

Giac [A]
time = 2.01, size = 47, normalized size = 0.59

$$\frac{1}{98304}\left(8\left(16\left(8\left(32\left(8\,x-15\right)x+243\right)x-27\right)x-405\right)x-3645\right)\sqrt{-4\,x^2+3\,x}+\frac{3645}{131072}\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/98304*(8*(16*(8*(32*(8*x - 15)*x + 243)*x - 27)*x - 405)*x - 3645)*sqrt(-
4*x^2 + 3*x) + 3645/131072*arcsin(8/3*x - 1)
```

Mupad [B]
time = 0.24, size = 63, normalized size = 0.80

$$\frac{3645\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{131072}+\frac{15\,\left(4\,x-\frac{3}{2}\right)\,{\left(3\,x-4\,x^2\right)}^{3/2}}{512}+\frac{\left(4\,x-\frac{3}{2}\right)\,{\left(3\,x-4\,x^2\right)}^{5/2}}{24}+\frac{405\,\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2)^(5/2),x)
```

```
[Out] (3645*asin((8*x)/3 - 1))/131072 + (15*(4*x - 3/2)*(3*x - 4*x^2)^(3/2))/512
+ ((4*x - 3/2)*(3*x - 4*x^2)^(5/2))/24 + (405*(x/2 - 3/16)*(3*x - 4*x^2)^(1
/2))/2048
```

3.8 $\int (3x-4x^2)^{3/2}\,dx$

Optimal. Leaf size=57

$$-\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

```
[Out] -1/32*(3-8*x)*(-4*x^2+3*x)^(3/2)+243/4096*arcsin(-1+8/3*x)-27/1024*(3-8*x)*
(-4*x^2+3*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {626, 633, 222}

$$-\frac{243\text{ArcSin}\left(1-\frac{8x}{3}\right)}{4096}-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (-27*(3 - 8*x)*Sqrt[3*x - 4*x^2])/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(3/2))/32
 - (243*ArcSin[1 - (8*x)/3])/4096
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(3x-4x^2\right)^{3/2}\,dx &= -\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}+\frac{27}{64}\int \sqrt{3x-4x^2}\,dx\\ &= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}+\frac{243\int \frac{1}{\sqrt{3x-4x^2}}\,dx}{2048}\\ &= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{81\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,\right.}{4096}\\ &= -\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 82, normalized size = 1.44

$$\frac{\sqrt{-x(-3+4x)}\left(-2\sqrt{x}\sqrt{-3+4x}\left(81+72x-1152x^2+1024x^3\right)+243\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)\right)}{2048\sqrt{x}\sqrt{-3+4x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (Sqrt[-(x*(-3 + 4*x))]*(-2*Sqrt[x]*Sqrt[-3 + 4*x]*(81 + 72*x - 1152*x^2 + 1
024*x^3) + 243*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]]))/(2048*Sqrt[x]*Sqrt[-3 + 4
*x])
```

Maple [A]
time = 0.41, size = 46, normalized size = 0.81

method	result
risch	$\frac{(1024x^3-1152x^2+72x+81)x(-3+4x)}{1024\sqrt{-x(-3+4x)}}+\frac{243\arcsin\left(-1+\frac{8x}{3}\right)}{4096}$
default	$-\frac{(3-8x)\left(-4x^2+3x\right)^{\frac{3}{2}}}{32}+\frac{243\arcsin\left(-1+\frac{8x}{3}\right)}{4096}-\frac{27(3-8x)\sqrt{-4x^2+3x}}{1024}$
meijerg	$-\frac{243i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{3}\,\left(\frac{5120}{27}x^3-\frac{640}{3}x^2+\frac{40}{3}x+15\right)\sqrt{-\frac{4x}{3}+1}}{360}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2\sqrt{3}\,\sqrt{x}}{3}\right)}{16}\right)}{128\sqrt{\pi}}$
trager	$\left(-x^3+\frac{9}{8}x^2-\frac{9}{128}x-\frac{81}{1024}\right)\sqrt{-4x^2+3x}-\frac{243\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(8x\,\mathrm{RootOf}\left(\_Z^2+1\right)+4\sqrt{-4x^2+3x}-\right.}{4096}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(3-8*x)*(-4*x^2+3*x)^(3/2)+243/4096*arcsin(-1+8/3*x)-27/1024*(3-8*x)*
(-4*x^2+3*x)^(1/2)
```

Maxima [A]
time = 0.52, size = 63, normalized size = 1.11

$$\frac{1}{4}\left(-4\,x^{2}+3\,x\right)^{\frac{3}{2}}x-\frac{3}{32}\left(-4\,x^{2}+3\,x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{-4\,x^{2}+3\,x}\,x-\frac{81}{1024}\sqrt{-4\,x^{2}+3\,x}-\frac{243}{4096}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-4*x^2 + 3*x)^(3/2)*x - 3/32*(-4*x^2 + 3*x)^(3/2) + 27/128*sqrt(-4*x^2
 + 3*x)*x - 81/1024*sqrt(-4*x^2 + 3*x) - 243/4096*arcsin(-8/3*x + 1)
```

Fricas [A]
time = 1.63, size = 48, normalized size = 0.84

$$-\frac{1}{1024}\left(1024\,x^{3}-1152\,x^{2}+72\,x+81\right)\sqrt{-4\,x^{2}+3\,x}-\frac{243}{2048}\arctan\left(\frac{\sqrt{-4\,x^{2}+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(1024*x^3 - 1152*x^2 + 72*x + 81)*sqrt(-4*x^2 + 3*x) - 243/2048*arc
tan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-4x^{2}+3x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(3/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(3/2), x)
```

Giac [A]
time = 1.81, size = 37, normalized size = 0.65

$$-\frac{1}{1024}\left(8\left(16\left(8\,x-9\right)x+9\right)x+81\right)\sqrt{-4\,x^{2}+3\,x}+\frac{243}{4096}\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1024*(8*(16*(8*x - 9)*x + 9)*x + 81)*sqrt(-4*x^2 + 3*x) + 243/4096*arcsi
n(8/3*x - 1)
```

Mupad [B]
time = 0.11, size = 45, normalized size = 0.79

$$\frac{243\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{4096}+\frac{\left(4\,x-\frac{3}{2}\right)\,{\left(3\,x-4\,x^2\right)}^{3/2}}{16}+\frac{27\,\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2)^(3/2),x)
```

```
[Out] (243*asin((8*x)/3 - 1))/4096 + ((4*x - 3/2)*(3*x - 4*x^2)^(3/2))/16 + (27*(
x/2 - 3/16)*(3*x - 4*x^2)^(1/2))/64
```

3.9 $\int \sqrt{3x-4x^2}\, dx$

Optimal. Leaf size=35

$$-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

```
[Out] 9/64*arcsin(-1+8/3*x)-1/16*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {626, 633, 222}

$$-\frac{9}{64}\text{ArcSin}\left(1-\frac{8x}{3}\right)-\frac{1}{16}\sqrt{3x-4x^2}\,(3-8x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x - 4*x^2],x]
```

```
[Out] -1/16*((3 - 8*x)*Sqrt[3*x - 4*x^2]) - (9*ArcSin[1 - (8*x)/3])/64
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{3x-4x^2}\,dx = -\frac{1}{16}(3-8x)\sqrt{3x-4x^2}+\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}\,dx$$
$$= -\frac{1}{16}(3-8x)\sqrt{3x-4x^2}-\frac{3}{64}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,3-8x\right)$$
$$= -\frac{1}{16}(3-8x)\sqrt{3x-4x^2}-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 1.57

$$\frac{1}{32}\sqrt{-x(-3+4x)}\left(-6+16x+\frac{9\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)}{\sqrt{x}\sqrt{-3+4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x - 4*x^2],x]
```

```
[Out] (Sqrt[-(x*(-3 + 4*x))]*(-6 + 16*x + (9*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]])/(S
qrt[x]*Sqrt[-3 + 4*x])))/32
```

Maple [A]
time = 0.41, size = 28, normalized size = 0.80

method	result
default	$\frac{9\arcsin\left(-1+\frac{8x}{3}\right)}{64}-\frac{(3-8x)\sqrt{-4x^2+3x}}{16}$
risch	$-\frac{(-3+8x)x(-3+4x)}{16\sqrt{-x(-3+4x)}}+\frac{9\arcsin\left(-1+\frac{8x}{3}\right)}{64}$
meijerg	$-\frac{9i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{3}\,(3-8x)\sqrt{-\frac{4x}{3}+1}}{9}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2\sqrt{3}\,\sqrt{x}}{3}\right)}{2}\right)}{16\sqrt{\pi}}$
trager	$\left(-\frac{3}{16}+\frac{x}{2}\right)\sqrt{-4x^2+3x}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-8x\,\mathrm{RootOf}\left(\_Z^2+1\right)+4\sqrt{-4x^2+3x}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 9/64*arcsin(-1+8/3*x)-1/16*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A]
time = 0.52, size = 36, normalized size = 1.03

$$\frac{1}{2}\sqrt{-4x^2+3x}\,x-\frac{3}{16}\sqrt{-4x^2+3x}-\frac{9}{64}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-4*x^2 + 3*x)*x - 3/16*sqrt(-4*x^2 + 3*x) - 9/64*arcsin(-8/3*x + 1
)
```

Fricas [A]
time = 1.60, size = 38, normalized size = 1.09

$$\frac{1}{16}\sqrt{-4x^2+3x}\,(8x-3)-\frac{9}{32}\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="fricas")
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) - 9/32*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-4x^2+3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(1/2),x)
[Out] Integral(sqrt(-4*x**2 + 3*x), x)
```

Giac [A]
time = 2.93, size = 27, normalized size = 0.77

$$\frac{1}{16}\sqrt{-4x^2+3x}\,(8x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="giac")
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.74

$$\frac{9\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{64}+\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2)^(1/2),x)
[Out] (9*asin((8*x)/3 - 1))/64 + (x/2 - 3/16)*(3*x - 4*x^2)^(1/2)
```

3.10 $\int \sqrt{6x-x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)$$

```
[Out] 9/2*arcsin(-1+1/3*x)-1/2*(3-x)*(-x^2+6*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {626, 633, 222}

$$-\frac{9}{2}\text{ArcSin}\left(1-\frac{x}{3}\right)-\frac{1}{2}\sqrt{6x-x^2}\,(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6*x - x^2],x]
```

```
[Out] -1/2*((3 - x)*Sqrt[6*x - x^2]) - (9*ArcSin[1 - x/3])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{6x-x^2}\,dx &= -\frac{1}{2}(3-x)\sqrt{6x-x^2}+\frac{9}{2}\int \frac{1}{\sqrt{6x-x^2}}\,dx\\
&= -\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx,x,6-2x\right)\\
&= -\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 1.23

$$\frac{1}{2}\sqrt{-((-6+x)x)}\left(-3+x-\frac{18\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-6+x}{x}}}\right)}{\sqrt{-6+x}\,\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6*x - x^2],x]
[Out] (Sqrt[-((-6 + x)*x)]*(-3 + x - (18*ArcTanh[1/Sqrt[(-6 + x)/x]])/(Sqrt[-6 +
x]*Sqrt[x])))/2
```

Maple [A]
time = 0.40, size = 28, normalized size = 0.80

method	result	size
risch	$-\frac{(x-3)x(x-6)}{2\sqrt{-x\,(x-6)}}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$	27
default	$-\frac{(-2x+6)\sqrt{-x^2+6x}}{4}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$	28
meijerg	$-\frac{18i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{6}\,(3-x)\sqrt{-\frac{x}{6}+1}}{36}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{6}\,\sqrt{x}}{6}\right)}{2}\right)}{\sqrt{\pi}}$	47

trager	$\left(\frac{x}{2}-\frac{3}{2}\right)\sqrt{-x^2+6x}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-x\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2+6x}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-2*x+6)*(-x^2+6*x)^(1/2)+9/2*arcsin(-1+1/3*x)
```

Maxima [A]
time = 0.55, size = 36, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+6\,x}\;x-\frac{3}{2}\sqrt{-x^2+6\,x}-\frac{9}{2}\,\arcsin\left(-\frac{1}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x + 1)
```

Fricas [A]
time = 1.31, size = 35, normalized size = 1.00

$$\frac{1}{2}\sqrt{-x^2+6\,x}\,(x-3)-9\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^2+6x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+6*x)**(1/2),x)
[Out] Integral(sqrt(-x**2 + 6*x), x)
```

Giac [A]
time = 1.58, size = 25, normalized size = 0.71

$$\frac{1}{2}\sqrt{-x^2+6\,x}\,(x-3)+\frac{9}{2}\,\arcsin\left(\frac{1}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.74

$$\frac{9\,\mathrm{asin}\left(\frac{x}{3}-1\right)}{2}+\left(\frac{x}{2}-\frac{3}{2}\right)\,\sqrt{6\,x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - x^2)^(1/2),x)
[Out] (9*asin(x/3 - 1))/2 + (x/2 - 3/2)*(6*x - x^2)^(1/2)
```

3.11 $\int \sqrt{5x-9x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{36}(5-18x)\sqrt{5x-9x^2}-\frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

```
[Out] 25/216*arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {626, 633, 222}

$$-\frac{25}{216}\text{ArcSin}\left(1-\frac{18x}{5}\right)-\frac{1}{36}\sqrt{5x-9x^2}\,(5-18x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5*x - 9*x^2],x]
```

```
[Out] -1/36*((5 - 18*x)*Sqrt[5*x - 9*x^2]) - (25*ArcSin[1 - (18*x)/5])/216
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule steps

$$\int \sqrt{5x-9x^2}\,dx = -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} + \frac{25}{72}\int \frac{1}{\sqrt{5x-9x^2}}\,dx$$
$$= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} - \frac{5}{216}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{25}}}\,dx, x, 5-18x\right)$$
$$= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} - \frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 1.57

$$\frac{1}{108}\sqrt{-x(-5+9x)}\left(-15+54x+\frac{25\log\left(-3\sqrt{x}+\sqrt{-5+9x}\right)}{\sqrt{x}\sqrt{-5+9x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5*x - 9*x^2],x]
[Out] (Sqrt[-(x*(-5 + 9*x))]*(-15 + 54*x + (25*Log[-3*Sqrt[x] + Sqrt[-5 + 9*x]])/
(Sqrt[x]*Sqrt[-5 + 9*x])))/108
```

Maple [A]
time = 0.40, size = 28, normalized size = 0.80

method	result
default	$\frac{25\arcsin\left(-1+\frac{18x}{5}\right)}{216} - \frac{(5-18x)\sqrt{-9x^2+5x}}{36}$
risch	$-\frac{(-5+18x)x(9x-5)}{36\sqrt{-x(9x-5)}} + \frac{25\arcsin\left(-1+\frac{18x}{5}\right)}{216}$
meijerg	$-\frac{25i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{5}\,\left(-\frac{54x}{5}+3\right)\sqrt{-\frac{9x}{5}+1}}{10}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{3\sqrt{5}\,\sqrt{x}}{5}\right)}{2}\right)}{54\sqrt{\pi}}$
trager	$\left(-\frac{5}{36}+\frac{x}{2}\right)\sqrt{-9x^2+5x} + \frac{25\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(-18x\,\text{RootOf}\left(\_Z^2+1\right)+6\sqrt{-9x^2+5x}+5\,\text{RootOf}\left(\_Z^2+1\right)\right)}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-9*x^2+5*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/216*arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^(1/2)
```

Maxima [A]
time = 0.52, size = 36, normalized size = 1.03

$$\frac{1}{2}\sqrt{-9x^2+5x}\,x-\frac{5}{36}\sqrt{-9x^2+5x}-\frac{25}{216}\arcsin\left(-\frac{18}{5}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-9*x^2 + 5*x)*x - 5/36*sqrt(-9*x^2 + 5*x) - 25/216*arcsin(-18/5*x
+ 1)
```

Fricas [A]
time = 1.51, size = 38, normalized size = 1.09

$$\frac{1}{36}\sqrt{-9x^2+5x}\,(18x-5)-\frac{25}{108}\arctan\left(\frac{\sqrt{-9x^2+5x}}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="fricas")
[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) - 25/108*arctan(1/3*sqrt(-9*x^2 + 5*x)/x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-9x^2+5x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x**2+5*x)**(1/2),x)
[Out] Integral(sqrt(-9*x**2 + 5*x), x)
```

Giac [A]
time = 1.68, size = 27, normalized size = 0.77

$$\frac{1}{36}\sqrt{-9x^2+5x}\,(18x-5)+\frac{25}{216}\arcsin\left(\frac{18}{5}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^2+5*x)^(1/2),x, algorithm="giac")
[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) + 25/216*arcsin(18/5*x - 1)
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.74

$$\frac{25\,\mathrm{asin}\left(\frac{18\,x}{5}-1\right)}{216}+\left(\frac{x}{2}-\frac{5}{36}\right)\,\sqrt{5\,x-9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 9*x^2)^(1/2),x)
[Out] (25*asin((18*x)/5 - 1))/216 + (x/2 - 5/36)*(5*x - 9*x^2)^(1/2)
```

3.12 $\int (x - x^2)^{3/2}\, dx$

Optimal. Leaf size=51

$$-\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{128}\sin^{-1}(1-2x)$$

```
[Out] -1/8*(1-2*x)*(-x^2+x)^(3/2)+3/128*arcsin(-1+2*x)-3/64*(1-2*x)*(-x^2+x)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$,
Rules used = {626, 633, 222}

$$-\frac{3}{128}\text{ArcSin}(1-2x)-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{64}(1-2x)\sqrt{x-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^2)^(3/2),x]
```

```
[Out] (-3*(1 - 2*x)*Sqrt[x - x^2])/64 - ((1 - 2*x)*(x - x^2)^(3/2))/8 - (3*ArcSin
[1 - 2*x])/128
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \left(x-x^2\right)^{3/2} dx = -\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2} + \frac{3}{16}\int \sqrt{x-x^2}\, dx$$
$$= -\frac{3}{64}(1-2x)\sqrt{x-x^2} - \frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2} + \frac{3}{128}\int \frac{1}{\sqrt{x-x^2}}\, dx$$
$$= -\frac{3}{64}(1-2x)\sqrt{x-x^2} - \frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2} - \frac{3}{128}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\, dx, x, 1-2x$$
$$= -\frac{3}{64}(1-2x)\sqrt{x-x^2} - \frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2} - \frac{3}{128}\sin^{-1}(1-2x)$$

Mathematica [A]
time = 0.13, size = 67, normalized size = 1.31

$$\frac{x(-3+x+26x^2-40x^3+16x^4)+6\sqrt{-1+x}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)}{64\sqrt{-((-1+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^2)^(3/2),x]
[Out] (x*(-3 + x + 26*x^2 - 40*x^3 + 16*x^4) + 6*Sqrt[-1 + x]*Sqrt[x]*ArcTanh[Sqr
t[-1 + x]/(-1 + Sqrt[x])])/(64*Sqrt[-((-1 + x)*x)])
```

Maple [A]
time = 0.41, size = 42, normalized size = 0.82

method	result
risch	$\frac{(16x^3-24x^2+2x+3)x(x-1)}{64\sqrt{-x(x-1)}} + \frac{3\arcsin(2x-1)}{128}$
default	$-\frac{(1-2x)(-x^2+x)^{\frac{3}{2}}}{8} + \frac{3\arcsin(2x-1)}{128} - \frac{3(1-2x)\sqrt{-x^2+x}}{64}$
meijerg	$-\frac{3i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,(80x^3-120x^2+10x+15)\sqrt{1-x}}{240}+\frac{i\sqrt{\pi}\,\arcsin(\sqrt{x})}{16}\right)}{4\sqrt{\pi}}$
trager	$\left(-\frac{1}{4}x^3+\frac{3}{8}x^2-\frac{1}{32}x-\frac{3}{64}\right)\sqrt{-x^2+x} - \frac{3\,\text{RootOf}(\_Z^2+1)\ln\left(2x\,\text{RootOf}(\_Z^2+1)+2\sqrt{-x^2+x}-\text{RootOf}(\_\right.}{128}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/8*(1-2*x)*(-x^2+x)^(3/2)+3/128*arcsin(2*x-1)-3/64*(1-2*x)*(-x^2+x)^(1/2)
```

Maxima [A]
time = 0.53, size = 55, normalized size = 1.08

$$\frac{1}{4}\left(-x^{2}+x\right)^{\frac{3}{2}}x-\frac{1}{8}\left(-x^{2}+x\right)^{\frac{3}{2}}+\frac{3}{32}\sqrt{-x^{2}+x}\,x-\frac{3}{64}\sqrt{-x^{2}+x}+\frac{3}{128}\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + x)^(3/2)*x - 1/8*(-x^2 + x)^(3/2) + 3/32*sqrt(-x^2 + x)*x - 3/6
4*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

Fricas [A]
time = 1.42, size = 43, normalized size = 0.84

$$-\frac{1}{64}\left(16\,x^{3}-24\,x^{2}+2\,x+3\right)\sqrt{-x^{2}+x}-\frac{3}{64}\arctan\left(\frac{\sqrt{-x^{2}+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 24*x^2 + 2*x + 3)*sqrt(-x^2 + x) - 3/64*arctan(sqrt(-x^2 +
x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^{2}+x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)**(3/2),x)
```

```
[Out] Integral((-x**2 + x)**(3/2), x)
```

Giac [A]
time = 1.91, size = 35, normalized size = 0.69

$$-\frac{1}{64}\left(2\left(4\left(2\,x-3\right)x+1\right)x+3\right)\sqrt{-x^{2}+x}+\frac{3}{128}\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*(4*(2*x - 3)*x + 1)*x + 3)*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

Mupad [B]
time = 0.19, size = 39, normalized size = 0.76

$$\frac{3\,\mathrm{asin}(2\,x-1)}{128}+\frac{3\,\sqrt{x-x^2}\,\left(\frac{x}{2}-\frac{1}{4}\right)}{16}+\frac{{\left(x-x^2\right)}^{3/2}\,\left(x-\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)^(3/2),x)
```

```
[Out] (3*asin(2*x - 1))/128 + (3*(x - x^2)^(1/2)*(x/2 - 1/4))/16 + ((x - x^2)^(3/
2)*(x - 1/2))/4
```

3.13 $\int \sqrt{4x+x^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}(2+x)\sqrt{4x+x^2}-4\tanh^{-1}\left(\frac{x}{\sqrt{4x+x^2}}\right)$$

```
[Out] -4*arctanh(x/(x^2+4*x)^(1/2))+1/2*(2+x)*(x^2+4*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {626, 634, 212}

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4*x + x^2],x]
```

```
[Out] ((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{4x+x^2}\,dx &= \frac{1}{2}(2+x)\sqrt{4x+x^2} - 2\int \frac{1}{\sqrt{4x+x^2}}\,dx \\ &= \frac{1}{2}(2+x)\sqrt{4x+x^2} - 4\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{4x+x^2}}\right) \\ &= \frac{1}{2}(2+x)\sqrt{4x+x^2} - 4\tanh^{-1}\left(\frac{x}{\sqrt{4x+x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 1.20

$$\frac{1}{2}\sqrt{x(4+x)}\left(2+x-\frac{8\tanh^{-1}\left(\sqrt{\frac{x}{4+x}}\right)}{\sqrt{x}\,\sqrt{4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4*x + x^2],x]
```

```
[Out] (Sqrt[x*(4 + x)]*(2 + x - (8*ArcTanh[Sqrt[x/(4 + x)]])/(Sqrt[x]*Sqrt[4 + x]
)))/2
```

Maple [A]
time = 0.39, size = 33, normalized size = 0.94

method	result	size
trager	$\left(\frac{x}{2}+1\right)\sqrt{x^2+4x} - 2\ln\left(2+x+\sqrt{x^2+4x}\right)$	32
default	$\frac{(2x+4)\sqrt{x^2+4x}}{4} - 2\ln\left(2+x+\sqrt{x^2+4x}\right)$	33
risch	$\frac{(2+x)x(x+4)}{2\sqrt{x(x+4)}} - 2\ln\left(2+x+\sqrt{x^2+4x}\right)$	33
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\left(\frac{3x}{2}+3\right)\sqrt{\frac{x}{4}+1}}{12}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{x}}{2}\right)}{2}\right)}{\sqrt{\pi}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*x+4)*(x^2+4*x)^(1/2)-2*ln(2+x+(x^2+4*x)^(1/2))
```

Maxima [A]
time = 0.30, size = 41, normalized size = 1.17

$$\frac{1}{2}\sqrt{x^2+4x}\,x+\sqrt{x^2+4x}-2\log\left(2x+2\sqrt{x^2+4x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 4*x)*x + sqrt(x^2 + 4*x) - 2*log(2*x + 2*sqrt(x^2 + 4*x) + 4
)
```

Fricas [A]
time = 1.72, size = 32, normalized size = 0.91

$$\frac{1}{2}\sqrt{x^2+4x}\,(x+2)+2\log\left(-x+\sqrt{x^2+4x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(-x + sqrt(x^2 + 4*x) - 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^2+4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x)**(1/2),x)
[Out] Integral(sqrt(x**2 + 4*x), x)
```

Giac [A]
time = 2.54, size = 33, normalized size = 0.94

$$\frac{1}{2}\sqrt{x^2+4x}\,(x+2)+2\log\left(\left|-x+\sqrt{x^2+4x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

Mupad [B]
time = 0.20, size = 29, normalized size = 0.83

$$\sqrt{x^2+4x}\,\left(\frac{x}{2}+1\right)-2\ln\left(x+\sqrt{x\,(x+4)}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + x^2)^(1/2),x)
[Out] (4*x + x^2)^(1/2)*(x/2 + 1) - 2*log(x + (x*(x + 4))^(1/2) + 2)
```

3.14 $\int \sqrt{-8x+x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2}(4-x)\sqrt{-8x+x^2}-16\tanh^{-1}\left(\frac{x}{\sqrt{-8x+x^2}}\right)$$

```
[Out] -16*arctanh(x/(x^2-8*x)^(1/2))-1/2*(4-x)*(x^2-8*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {626, 634, 212}

$$-\frac{1}{2}\sqrt{x^2-8x}\,(4-x)-16\tanh^{-1}\left(\frac{x}{\sqrt{x^2-8x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-8*x + x^2],x]
```

```
[Out] -1/2*((4 - x)*Sqrt[-8*x + x^2]) - 16*ArcTanh[x/Sqrt[-8*x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \sqrt{-8x+x^2}\,dx = -\frac{1}{2}(4-x)\sqrt{-8x+x^2} - 8\int \frac{1}{\sqrt{-8x+x^2}}\,dx$$
$$= -\frac{1}{2}(4-x)\sqrt{-8x+x^2} - 16\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-8x+x^2}}\right)$$
$$= -\frac{1}{2}(4-x)\sqrt{-8x+x^2} - 16\tanh^{-1}\left(\frac{x}{\sqrt{-8x+x^2}}\right)$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 1.14

$$\frac{1}{2}\sqrt{(-8+x)x}\left(-4+x-\frac{32\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-8+x}{x}}}\right)}{\sqrt{-8+x}\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-8*x + x^2],x]
[Out] (Sqrt[(-8 + x)*x]*(-4 + x - (32*ArcTanh[1/Sqrt[(-8 + x)/x]])/(Sqrt[-8 + x]*
Sqrt[x])))/2
```

Maple [A]
time = 0.41, size = 33, normalized size = 0.89

method	result	size
default	$\frac{(2x-8)\sqrt{x^2-8x}}{4} - 8\ln\left(x-4+\sqrt{x^2-8x}\right)$	33
risch	$\frac{(x-4)x(x-8)}{2\sqrt{x(x-8)}} - 8\ln\left(x-4+\sqrt{x^2-8x}\right)$	33
trager	$\left(\frac{x}{2}-2\right)\sqrt{x^2-8x} + 8\ln\left(4-x+\sqrt{x^2-8x}\right)$	34
meijerg	$-\frac{32i\sqrt{\operatorname{signum}(x-8)}\left(-\frac{i\sqrt{\pi}\sqrt{x}\sqrt{2}\left(-\frac{3x}{4}+3\right)\sqrt{-\frac{x}{8}+1}}{24}+\frac{i\sqrt{\pi}\arcsin\left(\frac{\sqrt{2}\sqrt{x}}{4}\right)}{2}\right)}{\sqrt{\pi}\sqrt{-\operatorname{signum}(x-8)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*x-8)*(x^2-8*x)^(1/2)-8*ln(x-4+(x^2-8*x)^(1/2))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 1.16

$$\frac{1}{2}\sqrt{x^2-8x}\,x-2\sqrt{x^2-8x}-8\log\left(2x+2\sqrt{x^2-8x}-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*x - 2*sqrt(x^2 - 8*x) - 8*log(2*x + 2*sqrt(x^2 - 8*x) -
 8)
```

Fricas [A]
time = 1.84, size = 32, normalized size = 0.86

$$\frac{1}{2}\sqrt{x^2-8x}\,(x-4)+8\log\left(-x+\sqrt{x^2-8x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(-x + sqrt(x^2 - 8*x) + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^2-8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 - 8*x), x)
```

Giac [A]
time = 1.84, size = 33, normalized size = 0.89

$$\frac{1}{2}\sqrt{x^2-8x}\,(x-4)+8\log\left(\left|-x+\sqrt{x^2-8x}+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-8*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(abs(-x + sqrt(x^2 - 8*x) + 4))
```

Mupad [B]
time = 0.11, size = 29, normalized size = 0.78

$$\left(\frac{x}{2}-2\right)\,\sqrt{x^2-8\,x}\,-8\,\ln\left(x+\sqrt{x\,\left(x-8\right)}\,-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 8*x)^(1/2),x)
```

```
[Out] (x/2 - 2)*(x^2 - 8*x)^(1/2) - 8*log(x + (x*(x - 8))^(1/2) - 4)
```

3.15 $\int \sqrt{-x+x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{4}(1-2x)\sqrt{-x+x^2}-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)$$

```
[Out] -1/4*arctanh(x/(x^2-x)^(1/2))-1/4*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {626, 634, 212}

$$-\frac{1}{4}\sqrt{x^2-x}\,(1-2x)-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x + x^2],x]
```

```
[Out] -1/4*((1 - 2*x)*Sqrt[-x + x^2]) - ArcTanh[x/Sqrt[-x + x^2]]/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \sqrt{-x+x^2}\,dx = -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{8}\int \frac{1}{\sqrt{-x+x^2}}\,dx$$
$$= -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)$$
$$= -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)$$

Mathematica [A]
time = 0.08, size = 50, normalized size = 1.28

$$\frac{1}{4}\sqrt{(-1+x)x}\left(-1+2x-\frac{2\tanh^{-1}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)}{\sqrt{-1+x}\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + x^2],x]
```

```
[Out] (Sqrt[(-1 + x)*x]*(-1 + 2*x - (2*ArcTanh[Sqrt[-1 + x]/(-1 + Sqrt[x])])/(Sqr
t[-1 + x]*Sqrt[x])))/4
```

Maple [A]
time = 0.41, size = 33, normalized size = 0.85

method	result	size
default	$\frac{(2x-1)\sqrt{x^2-x}}{4} - \frac{\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)}{8}$	33
risch	$\frac{(2x-1)x(x-1)}{4\sqrt{x(x-1)}} - \frac{\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)}{8}$	35
trager	$\left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x} - \frac{\ln\left(2x-1+2\sqrt{x^2-x}\right)}{8}$	36
meijerg	$-\frac{i\sqrt{\text{signum}(x-1)}\left(-\frac{i\sqrt{\pi}\sqrt{x}(-6x+3)\sqrt{1-x}}{6}+\frac{i\sqrt{\pi}\arcsin\left(\sqrt{x}\right)}{2}\right)}{2\sqrt{\pi}\sqrt{-\text{signum}(x-1)}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*x-1)*(x^2-x)^(1/2)-1/8*ln(x-1/2+(x^2-x)^(1/2))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 1.10

$$\frac{1}{2}\sqrt{x^2-x}\,x-\frac{1}{4}\sqrt{x^2-x}-\frac{1}{8}\log\left(2\,x+2\sqrt{x^2-x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 - x)*x - 1/4*sqrt(x^2 - x) - 1/8*log(2*x + 2*sqrt(x^2 - x) - 1
)
```

Fricas [A]
time = 1.60, size = 36, normalized size = 0.92

$$\frac{1}{4}\sqrt{x^2-x}\,(2\,x-1)+\frac{1}{8}\log\left(-2\,x+2\sqrt{x^2-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(-2*x + 2*sqrt(x^2 - x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^2-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 - x), x)
```

Giac [A]
time = 2.07, size = 37, normalized size = 0.95

$$\frac{1}{4}\sqrt{x^2-x}\,(2\,x-1)+\frac{1}{8}\log\left(\left|-2\,x+2\sqrt{x^2-x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
```

Mupad [B]
time = 0.20, size = 29, normalized size = 0.74

$$\sqrt{x^2-x}\,\left(\frac{x}{2}-\frac{1}{4}\right)-\frac{\ln\left(x+\sqrt{x\,(x-1)}-\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x)^(1/2),x)
[Out] (x^2 - x)^(1/2)*(x/2 - 1/4) - log(x + (x*(x - 1))^(1/2) - 1/2)/8
```

3.16 $\int \frac{1}{\left(bx+cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}+\frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}}-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}$$

```
[Out] -2/5*(2*c*x+b)/b^2/(c*x^2+b*x)^(5/2)+32/15*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(3/2
)-256/15*c^2*(2*c*x+b)/b^6/(c*x^2+b*x)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {628, 627}

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}+\frac{32c(b+2cx)}{15b^4\left(bx+cx^2\right)^{3/2}}-\frac{2(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) + (32*c*(b + 2*c*x))/(15*b^4*(
b*x + c*x^2)^(3/2)) - (256*c^2*(b + 2*c*x))/(15*b^6*Sqrt[b*x + c*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}} - \frac{(16c)\int \frac{1}{(bx+cx^2)^{5/2}}\,dx}{5b^2}$$
$$= -\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}} + \frac{32c(b+2cx)}{15b^4(bx+cx^2)^{3/2}} + \frac{(128c^2)\int \frac{1}{(bx+cx^2)^{3/2}}\,dx}{15b^4}$$
$$= -\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}} + \frac{32c(b+2cx)}{15b^4(bx+cx^2)^{3/2}} - \frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}$$

Mathematica [A]
time = 0.12, size = 70, normalized size = 0.84

$$-\frac{2(3b^5-10b^4cx+80b^3c^2x^2+480b^2c^3x^3+640bc^4x^4+256c^5x^5)}{15b^6(x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^3 + 640*b*c^4*x^4
+ 256*c^5*x^5))/(15*b^6*(x*(b + c*x))^(5/2))
```

Maple [A]
time = 0.42, size = 76, normalized size = 0.92

method	result	size
gosper	$-\frac{2x(cx+b)(256c^5x^5+640c^4x^4b+480b^2c^3x^3+80c^2x^2b^3-10cx\,b^4+3b^5)}{15b^6(c\,x^2+bx)^{\frac{7}{2}}}$	75
default	$-\frac{2(2cx+b)}{5b^2(c\,x^2+bx)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^2(c\,x^2+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{5b^2}$	76
trager	$-\frac{2(256c^5x^5+640c^4x^4b+480b^2c^3x^3+80c^2x^2b^3-10cx\,b^4+3b^5)\sqrt{c\,x^2+bx}}{15b^6(cx+b)^3x^3}$	79
risch	$-\frac{2(cx+b)(128c^2x^2-19bcx+3b^2)}{15b^6x^2\sqrt{x(cx+b)}}-\frac{2c^3(128c^2x^2+275bcx+150b^2)x}{15\sqrt{x(cx+b)}\,(c^2x^2+2bcx+b^2)b^6}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(2*c*x+b)/b^2/(c*x^2+b*x)^(5/2)-16/5*c/b^2*(-2/3*(2*c*x+b)/b^2/(c*x^2+
b*x)^(3/2)+16/3*c/b^4*(2*c*x+b)/(c*x^2+b*x)^(1/2))
```

Maxima [A]
time = 0.29, size = 111, normalized size = 1.34

$$-\frac{4\,cx}{5\,(cx^2+bx)^{\frac{5}{2}}b^2}+\frac{64\,c^2x}{15\,(cx^2+bx)^{\frac{3}{2}}b^4}-\frac{512\,c^3x}{15\,\sqrt{cx^2+bx}\,b^6}-\frac{2}{5\,(cx^2+bx)^{\frac{5}{2}}b}+\frac{32\,c}{15\,(cx^2+bx)^{\frac{3}{2}}b^3}-\frac{256\,c^2}{15\,\sqrt{cx^2+bx}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -4/5*c*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*x/((c*x^2 + b*x)^(3/2)*b^4)
- 512/15*c^3*x/(sqrt(c*x^2 + b*x)*b^6) - 2/5/((c*x^2 + b*x)^(5/2)*b) + 32/1
5*c/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c^2/(sqrt(c*x^2 + b*x)*b^5)
```

Fricas [A]
time = 1.96, size = 105, normalized size = 1.27

$$-\frac{2\,(256\,c^5x^5+640\,bc^4x^4+480\,b^2c^3x^3+80\,b^3c^2x^2-10\,b^4cx+3\,b^5)\sqrt{cx^2+bx}}{15\,(b^6c^3x^6+3\,b^7c^2x^5+3\,b^8cx^4+b^9x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(256*c^5*x^5 + 640*b*c^4*x^4 + 480*b^2*c^3*x^3 + 80*b^3*c^2*x^2 - 10*
b^4*c*x + 3*b^5)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c*x
^4 + b^9*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(bx+cx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(-7/2), x)
```

Giac [A]
time = 1.09, size = 74, normalized size = 0.89

$$-\frac{2\left(2\left(8\left(2\left(4\,x\left(\frac{2\,c^5x}{b^6}+\frac{5\,c^4}{b^5}\right)+\frac{15\,c^3}{b^4}\right)x+\frac{5\,c^2}{b^3}\right)x-\frac{5\,c}{b^2}\right)x+\frac{3}{b}\right)}{15\,(cx^2+bx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(2*(8*(2*(4*x*(2*c^5*x/b^6 + 5*c^4/b^5) + 15*c^3/b^4)*x + 5*c^2/b^3)*
x - 5*c/b^2)*x + 3/b)/(c*x^2 + b*x)^(5/2)
```

Mupad [B]
time = 0.27, size = 96, normalized size = 1.16

$$-\frac{6\,b^5+256\,b\,c^2\,{\left(c\,x^2+b\,x\right)}^2+512\,c^3\,x\,{\left(c\,x^2+b\,x\right)}^2-32\,b^3\,c\,\left(c\,x^2+b\,x\right)+12\,b^4\,c\,x-64\,b^2\,c^2\,x\,\left(c\,x^2+b\,x\right)}{15\,b^6\,{\left(c\,x^2+b\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(7/2),x)
```

```
[Out] -(6*b^5 + 256*b*c^2*(b*x + c*x^2)^2 + 512*c^3*x*(b*x + c*x^2)^2 - 32*b^3*c*
(b*x + c*x^2) + 12*b^4*c*x - 64*b^2*c^2*x*(b*x + c*x^2))/(15*b^6*(b*x + c*x
^2)^(5/2))
```

3.17 $\int \frac{1}{\sqrt{3ix+4x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

```
[Out] -1/2*I*arcsin(-1+8/3*I*x)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {633, 221}

$$\frac{1}{2}i\text{ArcSin}\left(1-\frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] (I/2)*ArcSin[1 - ((8*I)/3)*x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3ix+4x^2}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}}\,dx, x, 3i+8x\right)\\ &= \frac{1}{2}i\sin^{-1}\left(1-\frac{8ix}{3}\right)\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

time = 0.03, size = 51, normalized size = 3.19

$$-\frac{\sqrt{x}\,\sqrt{3i+4x}\,\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{\sqrt{x(3i+4x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] -((Sqrt[x]*Sqrt[3*I + 4*x]*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/Sqrt[x*(3*I +
 4*x)])
```

Maple [A]
time = 0.43, size = 10, normalized size = 0.62

method	result	size
default	$\frac{\operatorname{arcsinh}\left(i+\frac{8x}{3}\right)}{2}$	10
trager	$-\frac{\ln\left(-440x-144-165i-192i\sqrt{4x^2+3ix}+384ix+220\sqrt{4x^2+3ix}\right)}{2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*I*x+4*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(I+8/3*x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.
time = 0.49, size = 21, normalized size = 1.31

$$\frac{1}{2}\log\left(8\,x+4\,\sqrt{4\,x^2+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.
time = 1.35, size = 19, normalized size = 1.19

$$-\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4x^2+3ix}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**2 + 3*I*x), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(8) = 16.
time = 1.58, size = 110, normalized size = 6.88

$$\frac{1}{32}\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}\,x}\,(8\,x+3i)\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-\frac{9}{64}\log\left(2\,\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}\,x}\,\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-8\,x-3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(16
*x^4 + 9*x^2)) + 1) - 9/64*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/
(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

Mupad [B]
time = 0.28, size = 19, normalized size = 1.19

$$\frac{\ln\left(x+\frac{\sqrt{x\,\left(4\,x+3\mathrm{i}\right)}}{2}+\frac{3}{8}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*3i + 4*x^2)^(1/2),x)
```

```
[Out] log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8)/2
```

3.18 $\int \frac{1}{\left(3ix+4x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=26

$$\frac{2(3i+8x)}{9\sqrt{3ix+4x^2}}$$

```
[Out] 2/9*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{627\}$

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-3/2),x]
[Out] (2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(3ix+4x^2\right)^{3/2}} \, dx = \frac{2(3i+8x)}{9\sqrt{3ix+4x^2}}$$

Mathematica [A]
time = 0.05, size = 24, normalized size = 0.92

$$\frac{2(3i+8x)}{9\sqrt{x(3i+4x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-3/2),x]
[Out] (2*(3*I + 8*x))/(9*Sqrt[x*(3*I + 4*x)])
```

Maple [A]
time = 0.38, size = 21, normalized size = 0.81

method	result	size
risch	$\frac{\frac{2i}{3}+\frac{16x}{9}}{\sqrt{x\,(3i+4x)}}$	19
default	$\frac{\frac{2i}{3}+\frac{16x}{9}}{\sqrt{4x^{2}+3ix}}$	21
trager	$\frac{\left(-\frac{14}{225}+\frac{16i}{75}\right)(24ix+32x+12i-9)\sqrt{4x^{2}+3ix}}{x(12ix-16x-12i-9)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*I*x+4*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 28, normalized size = 1.08

$$\frac{16\,x}{9\,\sqrt{4\,x^{2}+3i\,x}}+\frac{2i}{3\,\sqrt{4\,x^{2}+3i\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/9*x/sqrt(4*x^2 + 3*I*x) + 2/3*I/sqrt(4*x^2 + 3*I*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.
time = 2.00, size = 39, normalized size = 1.50

$$\frac{2\left(16\,x^{2}+\sqrt{4\,x^{2}+3i\,x}\,(8\,x+3i)+12i\,x\right)}{9\,(4\,x^{2}+3i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(16*x^2 + sqrt(4*x^2 + 3*I*x)*(8*x + 3*I) + 12*I*x)/(4*x^2 + 3*I*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(4x^{2}+3ix)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-3/2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(18) = 36$.
time = 1.90, size = 64, normalized size = 2.46

$$\frac{\sqrt{8\,x^2 + 2\,\sqrt{16\,x^2 + 9}\,x}\,(8\,x + 3i)\left(\frac{3i\,x}{4\,x^2 + \sqrt{16\,x^4 + 9\,x^2}} + 1\right)}{9\,(4\,x^2 + 3i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(16*
x^4 + 9*x^2)) + 1)/(4*x^2 + 3*I*x)
```

Mupad [B]
time = 0.05, size = 20, normalized size = 0.77

$$\frac{16\,x + 6\mathrm{i}}{9\,\sqrt{4\,x^2 + x\,3\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*3i + 4*x^2)^(3/2),x)
```

```
[Out] (16*x + 6i)/(9*(x*3i + 4*x^2)^(1/2))
```

3.19 $\int \frac{1}{(3ix+4x^2)^{5/2}} dx$

Optimal. Leaf size=53

$$\frac{2(3i+8x)}{27(3ix+4x^2)^{3/2}}+\frac{64(3i+8x)}{243\sqrt{3ix+4x^2}}$$

```
[Out] 2/27*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+64/243*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {628, 627}

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27(4x^2+3ix)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-5/2),x]
```

```
[Out] (2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[(
3*I)*x + 4*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx &= \frac{2(3i+8x)}{27(3ix+4x^2)^{3/2}}+\frac{32}{27}\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx\\ &= \frac{2(3i+8x)}{27(3ix+4x^2)^{3/2}}+\frac{64(3i+8x)}{243\sqrt{3ix+4x^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 36, normalized size = 0.68

$$\frac{54i - 432x + 2304ix^2 + 2048x^3}{243(x(3i + 4x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-5/2),x]
[Out] (54*I - 432*x + (2304*I)*x^2 + 2048*x^3)/(243*(x*(3*I + 4*x))^(3/2))
```

Maple [A]
time = 0.40, size = 42, normalized size = 0.79

method	result	size
risch	$\frac{\frac{2048}{243}x^3+\frac{256}{27}ix^2-\frac{16}{9}x+\frac{2}{9}i}{x(3i+4x)\sqrt{x\left(3i+4x\right)}}$	41
default	$\frac{\frac{2i}{9}+\frac{16x}{27}}{(4x^2+3ix)^{\frac{3}{2}}}+\frac{\frac{64i}{81}+\frac{512x}{243}}{\sqrt{4x^2+3ix}}$	42
trager	$\frac{\left(\frac{88}{151875}+\frac{26i}{16875}\right)\left(-76800ix^3-102400x^3-115200ix^2+86400x^2+16200ix+21600x-2700i+2025\right)\sqrt{4x^2+3ix}}{(12ix-16x-12i-9)^2x^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*I*x+4*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/27*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+64/243*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 55, normalized size = 1.04

$$\frac{512\,x}{243\,\sqrt{4\,x^2+3i\,x}}+\frac{64i}{81\,\sqrt{4\,x^2+3i\,x}}+\frac{16\,x}{27\,(4\,x^2+3i\,x)^{\frac{3}{2}}}+\frac{2i}{9\,(4\,x^2+3i\,x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
[Out] 512/243*x/sqrt(4*x^2 + 3*I*x) + 64/81*I/sqrt(4*x^2 + 3*I*x) + 16/27*x/(4*x^
2 + 3*I*x)^(3/2) + 2/9*I/(4*x^2 + 3*I*x)^(3/2)
```

Fricas [A]
time = 1.44, size = 63, normalized size = 1.19

$$\frac{2\left(2048\,x^4+3072i\,x^3-1152\,x^2+(1024\,x^3+1152i\,x^2-216\,x+27i)\sqrt{4\,x^2+3i\,x}\right)}{243\,(16\,x^4+24i\,x^3-9\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/243*(2048*x^4 + 3072*I*x^3 - 1152*x^2 + (1024*x^3 + 1152*I*x^2 - 216*x +
27*I)*sqrt(4*x^2 + 3*I*x))/(16*x^4 + 24*I*x^3 - 9*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4x^2+3ix)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-5/2), x)
```

Giac [A]
time = 2.10, size = 74, normalized size = 1.40

$$\frac{(8\,(16\,(8\,x+9i)x-27)x+27i)\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}\,x}\,\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)}{243\,(4\,x^2+3i\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/243*(8*(16*(8*x + 9*I)*x - 27)*x + 27*I)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*
x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1)/(4*x^2 + 3*I*x)^2
```

Mupad [B]
time = 0.12, size = 31, normalized size = 0.58

$$\frac{(16\,x+6\mathrm{i})\,\left(128\,x^2+x\,96\mathrm{i}+9\right)}{243\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*3i + 4*x^2)^(5/2),x)
```

```
[Out] ((16*x + 6i)*(x*96i + 128*x^2 + 9))/(243*(x*3i + 4*x^2)^(3/2))
```

3.20 $\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(3i+8x)}{45\,(3ix+4x^2)^{5/2}}+\frac{128(3i+8x)}{1215\,(3ix+4x^2)^{3/2}}+\frac{4096(3i+8x)}{10935\sqrt{3ix+4x^2}}$$

```
[Out] 2/45*(3*I+8*x)/(3*I*x+4*x^2)^(5/2)+128/1215*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+4
096/10935*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {628, 627}

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}}+\frac{128(8x+3i)}{1215\,(4x^2+3ix)^{3/2}}+\frac{2(8x+3i)}{45\,(4x^2+3ix)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^(5/2)) + (128*(3*I + 8*x))/(1215*((3*
I)*x + 4*x^2)^(3/2)) + (4096*(3*I + 8*x))/(10935*Sqrt[(3*I)*x + 4*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \frac{2(3i+8x)}{45\,(3ix+4x^2)^{5/2}} + \frac{64}{45}\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx$$
$$= \frac{2(3i+8x)}{45\,(3ix+4x^2)^{5/2}} + \frac{128(3i+8x)}{1215\,(3ix+4x^2)^{3/2}} + \frac{2048\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx}{1215}$$
$$= \frac{2(3i+8x)}{45\,(3ix+4x^2)^{5/2}} + \frac{128(3i+8x)}{1215\,(3ix+4x^2)^{3/2}} + \frac{4096(3i+8x)}{10935\sqrt{3ix+4x^2}}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 0.61

$$\frac{1458i - 6480x - 69120ix^2 - 552960x^3 + 983040ix^4 + 524288x^5}{10935(x(3i+4x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (1458*I - 6480*x - (69120*I)*x^2 - 552960*x^3 + (983040*I)*x^4 + 524288*x^5
)/(10935*(x*(3*I + 4*x))^(5/2))
```

Maple [A]
time = 0.42, size = 62, normalized size = 0.78

method	result
risch	$\frac{\frac{524288}{10935}x^5+\frac{65536}{729}ix^4-\frac{4096}{81}x^3-\frac{512}{81}ix^2-\frac{16}{27}x+\frac{2}{15}i}{x^2(3i+4x)^2\sqrt{x\,(3i+4x)}}$
default	$\frac{\frac{2i}{15}+\frac{16x}{45}}{(4x^2+3ix)^{\frac{5}{2}}}+\frac{\frac{128i}{405}+\frac{1024x}{1215}}{(4x^2+3ix)^{\frac{3}{2}}}+\frac{\frac{4096i}{3645}+\frac{32768x}{10935}}{\sqrt{4x^2+3ix}}$
trager	$\frac{\left(\frac{1054}{4271484375}+\frac{224i}{1423828125}\right)(12288000000ix^5+16384000000x^5+30720000000ix^4-23040000000x^4-12960000000ix^3-17280000000x^3-2}{(12ix-16x-12i-9)^3x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*I*x+4*x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(3*I+8*x)/(3*I*x+4*x^2)^(5/2)+128/1215*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+4
096/10935*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 82, normalized size = 1.04

$$\frac{32768\,x}{10935\,\sqrt{4\,x^2+3i\,x}} + \frac{4096i}{3645\,\sqrt{4\,x^2+3i\,x}} + \frac{1024\,x}{1215\,(4\,x^2+3i\,x)^{\frac{3}{2}}} + \frac{128i}{405\,(4\,x^2+3i\,x)^{\frac{3}{2}}} + \frac{16\,x}{45\,(4\,x^2+3i\,x)^{\frac{5}{2}}} + \frac{2i}{15\,(4\,x^2+3i\,x)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 32768/10935*x/sqrt(4*x^2 + 3*I*x) + 4096/3645*I/sqrt(4*x^2 + 3*I*x) + 1024/
1215*x/(4*x^2 + 3*I*x)^(3/2) + 128/405*I/(4*x^2 + 3*I*x)^(3/2) + 16/45*x/(4
*x^2 + 3*I*x)^(5/2) + 2/15*I/(4*x^2 + 3*I*x)^(5/2)
```

Fricas [A]
time = 1.65, size = 83, normalized size = 1.05

$$\frac{2\left(524288\,x^6+1179648i\,x^5-884736\,x^4-221184i\,x^3+\left(262144\,x^5+491520i\,x^4-276480\,x^3-34560i\,x^2-3240\,x+729i\right)\sqrt{4\,x^2+3i\,x}\right)}{10935\left(64\,x^6+144i\,x^5-108\,x^4-27i\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/10935*(524288*x^6 + 1179648*I*x^5 - 884736*x^4 - 221184*I*x^3 + (262144*x
^5 + 491520*I*x^4 - 276480*x^3 - 34560*I*x^2 - 3240*x + 729*I)*sqrt(4*x^2 +
 3*I*x))/(64*x^6 + 144*I*x^5 - 108*x^4 - 27*I*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4x^2+3ix\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(7/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-7/2), x)
```

Giac [A]
time = 1.78, size = 84, normalized size = 1.06

$$\frac{\left(8\left(32\left(8\left(16\left(8\,x+15i\right)x-135\right)x-135i\right)x-405\right)x+729i\right)\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}\,x}\,\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)}{10935\left(4\,x^2+3i\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/10935*(8*(32*(8*(16*(8*x + 15*I)*x - 135)*x - 135*I)*x - 405)*x + 729*I)*
sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) +
1)/(4*x^2 + 3*I*x)^3
```

Mupad [B]
time = 0.29, size = 40, normalized size = 0.51

$$-\frac{-524288\,x^5-x^4\,983040\mathrm{i}+552960\,x^3+x^2\,69120\mathrm{i}+6480\,x-1458\mathrm{i}}{10935\left(x\,\left(4\,x+3\mathrm{i}\right)\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*3i + 4*x^2)^(7/2),x)
```

```
[Out] -(6480*x + x^2*69120i + 552960*x^3 - x^4*983040i - 524288*x^5 - 1458i)/(109
35*(x*(4*x + 3i))^(5/2))
```

3.21 $\int \frac{1}{\sqrt{3x-4x^2}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

```
[Out] 1/2*arcsin(-1+8/3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {633, 222}

$$-\frac{1}{2}\text{ArcSin}\left(1-\frac{8x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3*x - 4*x^2],x]
```

```
[Out] -1/2*ArcSin[1 - (8*x)/3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3x-4x^2}}\,dx &= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 3-8x\right)\right)\\ &= -\frac{1}{2}\sin^{-1}\left(1-\frac{8x}{3}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(12) = 24.

time = 0.03, size = 46, normalized size = 3.83

$$-\frac{\sqrt{x}\,\sqrt{-3+4x}\,\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)}{\sqrt{-x(-3+4x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3*x - 4*x^2],x]
```

```
[Out] -((Sqrt[x]*Sqrt[-3 + 4*x]*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]])/Sqrt[-(x*(-3 +
4*x))])
```

Maple [A]
time = 0.39, size = 9, normalized size = 0.75

method	result	size
default	$\frac{\arcsin\left(-1+\frac{8x}{3}\right)}{2}$	9
meijerg	$\arcsin\left(\frac{2\sqrt{3}\,\sqrt{x}}{3}\right)$	10
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(8x\,\mathrm{RootOf}\left(\_Z^2+1\right)+4\sqrt{-4x^2+3x}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(-1+8/3*x)
```

Maxima [A]
time = 0.54, size = 8, normalized size = 0.67

$$-\frac{1}{2}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-8/3*x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.
time = 1.33, size = 19, normalized size = 1.58

$$-\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^2+3x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**2 + 3*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(8) = 16.
time = 3.70, size = 27, normalized size = 2.25

$$\frac{1}{16}\sqrt{-4\,x^2+3\,x}\,(8\,x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

Mupad [**B**]
time = 0.11, size = 8, normalized size = 0.67

$$\frac{\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(1/2),x)
```

```
[Out] asin((8*x)/3 - 1)/2
```

3.22 $\int \frac{1}{(3x-4x^2)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

```
[Out] -2/9*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{627\}$

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-3/2),x]
[Out] (-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = -\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Mathematica [A]
time = 0.05, size = 21, normalized size = 0.95

$$\frac{2(-3+8x)}{9\sqrt{-x(-3+4x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-3/2),x]
[Out] (2*(-3 + 8*x))/(9*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A]
time = 0.38, size = 19, normalized size = 0.86

method	result	size
default	$-\frac{2(3-8x)}{9\sqrt{-4x^2+3x}}$	19
meijerg	$-\frac{2\sqrt{3}\,(1-\frac{8x}{3})}{9\sqrt{x}\,\sqrt{-\frac{4x}{3}+1}}$	21
gosper	$-\frac{2x(-3+4x)(-3+8x)}{9(-4x^2+3x)^{\frac{3}{2}}}$	25
trager	$-\frac{2(-3+8x)\sqrt{-4x^2+3x}}{9x(-3+4x)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

Maxima [A]
time = 0.29, size = 28, normalized size = 1.27

$$\frac{16\,x}{9\,\sqrt{-4\,x^2+3\,x}}-\frac{2}{3\,\sqrt{-4\,x^2+3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/9*x/sqrt(-4*x^2 + 3*x) - 2/3/sqrt(-4*x^2 + 3*x)
```

Fricas [A]
time = 1.65, size = 29, normalized size = 1.32

$$-\frac{2\,\sqrt{-4\,x^2+3\,x}\,(8\,x-3)}{9\,(4\,x^2-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-4x^2+3x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(3/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-3/2), x)
```

Giac [A]
time = 1.67, size = 29, normalized size = 1.32

$$-\frac{2\sqrt{-4x^2+3x}\,(8x-3)}{9\,(4x^2-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

Mupad [B]
time = 0.14, size = 18, normalized size = 0.82

$$\frac{16x-6}{9\sqrt{3x-4x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(3/2),x)
```

```
[Out] (16*x - 6)/(9*(3*x - 4*x^2)^(1/2))
```

3.23 $\int \frac{1}{(3x-4x^2)^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}$$

```
[Out] -2/27*(3-8*x)/(-4*x^2+3*x)^(3/2)-64/243*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {628, 627}

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-5/2),x]
```

```
[Out] (-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqrt[3*x - 4*
x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(3x-4x^2\right)^{5/2}}\,dx &= -\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}+\frac{32}{27}\int \frac{1}{\left(3x-4x^2\right)^{3/2}}\,dx\\ &= -\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.69

$$-\frac{54+432x-2304x^2+2048x^3}{243(-x(-3+4x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-5/2),x]
[Out] -1/243*(54 + 432*x - 2304*x^2 + 2048*x^3)/(-(x*(-3 + 4*x)))^(3/2)
```

Maple [A]
time = 0.39, size = 38, normalized size = 0.84

method	result	size
meijerg	$-\frac{2\sqrt{3}\,\left(\frac{1024}{27}x^3-\frac{128}{3}x^2+8x+1\right)}{81x^{\frac{3}{2}}\left(-\frac{4x}{3}+1\right)^{\frac{3}{2}}}$	31
gosper	$\frac{2x(-3+4x)(1024x^3-1152x^2+216x+27)}{243(-4x^2+3x)^{\frac{5}{2}}}$	35
default	$-\frac{2(3-8x)}{27(-4x^2+3x)^{\frac{3}{2}}}-\frac{64(3-8x)}{243\sqrt{-4x^2+3x}}$	38
trager	$-\frac{2(1024x^3-1152x^2+216x+27)\sqrt{-4x^2+3x}}{243(-3+4x)^2x^2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/27*(3-8*x)/(-4*x^2+3*x)^(3/2)-64/243*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

Maxima [A]
time = 0.31, size = 55, normalized size = 1.22

$$\frac{512\,x}{243\,\sqrt{-4\,x^2+3\,x}}-\frac{64}{81\,\sqrt{-4\,x^2+3\,x}}+\frac{16\,x}{27\,(-4\,x^2+3\,x)^{\frac{3}{2}}}-\frac{2}{9\,(-4\,x^2+3\,x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="maxima")
[Out] 512/243*x/sqrt(-4*x^2 + 3*x) - 64/81/sqrt(-4*x^2 + 3*x) + 16/27*x/(-4*x^2 +
 3*x)^(3/2) - 2/9/(-4*x^2 + 3*x)^(3/2)
```

Fricas [A]
time = 1.38, size = 46, normalized size = 1.02

$$-\frac{2\,(1024\,x^3-1152\,x^2+216\,x+27)\sqrt{-4\,x^2+3\,x}}{243\,(16\,x^4-24\,x^3+9\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/243*(1024*x^3 - 1152*x^2 + 216*x + 27)*sqrt(-4*x^2 + 3*x)/(16*x^4 - 24*x
^3 + 9*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-4x^2+3x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-5/2), x)
```

Giac [**A**]
time = 2.58, size = 39, normalized size = 0.87

$$-\frac{2\,(8\,(16\,(8\,x-9)x+27)x+27)\sqrt{-4\,x^2+3\,x}}{243\,(4\,x^2-3\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/243*(8*(16*(8*x - 9)*x + 27)*x + 27)*sqrt(-4*x^2 + 3*x)/(4*x^2 - 3*x)^2
```

Mupad [**B**]
time = 0.03, size = 28, normalized size = 0.62

$$\frac{(16\,x-6)\,\left(-128\,x^2+96\,x+9\right)}{243\,{\left(3\,x-4\,x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(5/2),x)
```

```
[Out] ((16*x - 6)*(96*x - 128*x^2 + 9))/(243*(3*x - 4*x^2)^(3/2))
```

3.24 $\int \frac{1}{(3x-4x^2)^{7/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}-\frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}}-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}$$

```
[Out] -2/45*(3-8*x)/(-4*x^2+3*x)^(5/2)-128/1215*(3-8*x)/(-4*x^2+3*x)^(3/2)-4096/1
0935*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {628, 627}

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}-\frac{128(3-8x)}{1215\left(3x-4x^2\right)^{3/2}}-\frac{2(3-8x)}{45\left(3x-4x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (-2*(3 - 8*x))/(45*(3*x - 4*x^2)^(5/2)) - (128*(3 - 8*x))/(1215*(3*x - 4*x^
2)^(3/2)) - (4096*(3 - 8*x))/(10935*Sqrt[3*x - 4*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = -\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}} + \frac{64}{45}\int \frac{1}{(3x-4x^2)^{5/2}}\,dx$$
$$= -\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}} - \frac{128(3-8x)}{1215\,(3x-4x^2)^{3/2}} + \frac{2048\int \frac{1}{(3x-4x^2)^{3/2}}\,dx}{1215}$$
$$= -\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}} - \frac{128(3-8x)}{1215\,(3x-4x^2)^{3/2}} - \frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 0.61

$$\frac{2(-729-3240x-34560x^2+276480x^3-491520x^4+262144x^5)}{10935(-x(-3+4x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (2*(-729 - 3240*x - 34560*x^2 + 276480*x^3 - 491520*x^4 + 262144*x^5))/(109
35*(-(x*(-3 + 4*x)))^(5/2))
```

Maple [A]
time = 0.38, size = 56, normalized size = 0.84

method	result	size
meijerg	$-\frac{2\sqrt{3}\,\left(-\frac{262144}{243}x^5+\frac{163840}{81}x^4-\frac{10240}{9}x^3+\frac{1280}{9}x^2+\frac{40}{3}x+3\right)}{1215x^{\frac{5}{2}}\left(-\frac{4x}{3}+1\right)^{\frac{5}{2}}}$	41
gosper	$-\frac{2x(-3+4x)\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)}{10935(-4x^2+3x)^{\frac{7}{2}}}$	45
trager	$-\frac{2\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)\sqrt{-4x^2+3x}}{10935(-3+4x)^3x^3}$	49
default	$-\frac{2(3-8x)}{45(-4x^2+3x)^{\frac{5}{2}}}-\frac{128(3-8x)}{1215(-4x^2+3x)^{\frac{3}{2}}}-\frac{4096(3-8x)}{10935\sqrt{-4x^2+3x}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+3*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(3-8*x)/(-4*x^2+3*x)^(5/2)-128/1215*(3-8*x)/(-4*x^2+3*x)^(3/2)-4096/1
0935*(3-8*x)/(-4*x^2+3*x)^(1/2)
```

Maxima [A]
time = 0.30, size = 82, normalized size = 1.22

$$\frac{32768\,x}{10935\,\sqrt{-4\,x^2+3\,x}} - \frac{4096}{3645\,\sqrt{-4\,x^2+3\,x}} + \frac{1024\,x}{1215\,(-4\,x^2+3\,x)^{\frac{3}{2}}} - \frac{128}{405\,(-4\,x^2+3\,x)^{\frac{3}{2}}} + \frac{16\,x}{45\,(-4\,x^2+3\,x)^{\frac{5}{2}}} - \frac{2}{15\,(-4\,x^2+3\,x)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 32768/10935*x/sqrt(-4*x^2 + 3*x) - 4096/3645/sqrt(-4*x^2 + 3*x) + 1024/1215
*x/(-4*x^2 + 3*x)^(3/2) - 128/405/(-4*x^2 + 3*x)^(3/2) + 16/45*x/(-4*x^2 +
3*x)^(5/2) - 2/15/(-4*x^2 + 3*x)^(5/2)
```

Fricas [A]
time = 1.76, size = 61, normalized size = 0.91

$$-\frac{2\,(262144\,x^5-491520\,x^4+276480\,x^3-34560\,x^2-3240\,x-729)\sqrt{-4\,x^2+3\,x}}{10935\,(64\,x^6-144\,x^5+108\,x^4-27\,x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/10935*(262144*x^5 - 491520*x^4 + 276480*x^3 - 34560*x^2 - 3240*x - 729)*
sqrt(-4*x^2 + 3*x)/(64*x^6 - 144*x^5 + 108*x^4 - 27*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-4x^2+3x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-7/2), x)
```

Giac [A]
time = 1.82, size = 49, normalized size = 0.73

$$-\frac{2\,(8\,(32\,(8\,(16\,(8\,x-15)x+135)x-135)x-405)x-729)\sqrt{-4\,x^2+3\,x}}{10935\,(4\,x^2-3\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/10935*(8*(32*(8*(16*(8*x - 15)*x + 135)*x - 135)*x - 405)*x - 729)*sqrt(
-4*x^2 + 3*x)/(4*x^2 - 3*x)^3
```

Mupad [B]
time = 0.20, size = 73, normalized size = 1.09

$$-\frac{6480\,x-9216\,x\,(3\,x-4\,x^2)-32768\,x\,(3\,x-4\,x^2)^2+12288\,(3\,x-4\,x^2)^2-13824\,x^2+1458}{(3\,x-4\,x^2)^{3/2}\,(32805\,x-43740\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(7/2),x)
```

```
[Out] -(6480*x - 9216*x*(3*x - 4*x^2) - 32768*x*(3*x - 4*x^2)^2 + 12288*(3*x - 4*
x^2)^2 - 13824*x^2 + 1458)/((3*x - 4*x^2)^(3/2)*(32805*x - 43740*x^2))
```

3.25 $\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin^{-1}(1-2bx)}{b}$$

```
[Out] arcsin(2*b*x-1)/b
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {633, 222}

$$-\frac{\text{ArcSin}(1-2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x - b^2*x^2],x]
```

```
[Out] -(ArcSin[1 - 2*b*x]/b)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{b^2}}}\,dx, x, b-2b^2x\right)}{b^2}\\ &= -\frac{\sin^{-1}(1-2bx)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.
time = 0.04, size = 57, normalized size = 4.75

$$-\frac{2\sqrt{x}\sqrt{-1+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{-1+bx}\right)}{\sqrt{b}\sqrt{-bx(-1+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x - b^2*x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[-1 + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[-1 + b*x]])/(Sqrt[
b]*Sqrt[-(b*x*(-1 + b*x))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(11) = 22.
time = 0.38, size = 35, normalized size = 2.92

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{b^2}\left(x-\frac{1}{2b}\right)}{\sqrt{-b^2x^2+bx}}\right)}{\sqrt{b^2}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b^2*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x)^(1/2))
```

Maxima [A]
time = 0.48, size = 21, normalized size = 1.75

$$-\frac{\arcsin\left(-\frac{2\,b^2x-b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*b^2*x - b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.
time = 1.27, size = 27, normalized size = 2.25

$$-\frac{2\,\arctan\left(\frac{\sqrt{-b^2x^2+bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-b^2*x^2 + b*x)/(b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b**2*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-b**2*x**2 + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.
time = 2.73, size = 41, normalized size = 3.42

$$\frac{1}{4}\sqrt{-b^2x^2+bx}\left(2\,x-\frac{1}{b}\right)-\frac{\arcsin\left(-2\,bx+1\right)\operatorname{sgn}(b)}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-b^2*x^2 + b*x)*(2*x - 1/b) - 1/8*arcsin(-2*b*x + 1)*sgn(b)/abs(b)
```

Mupad [B]
time = 0.31, size = 42, normalized size = 3.50

$$\frac{\ln\left(\frac{\frac{b}{2}-b^2\,x}{\sqrt{-b^2}}+\sqrt{b\,x-b^2\,x^2}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x - b^2*x^2)^(1/2),x)
```

```
[Out] log((b/2 - b^2*x)/(-b^2)^(1/2) + (b*x - b^2*x^2)^(1/2))/(-b^2)^(1/2)
```

3.26 $\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx$

Optimal. Leaf size=24

$$\frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{bx+b^2x^2}}\right)}{b}$$

```
[Out] 2*arctanh(b*x/(b^2*x^2+b*x)^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {634, 212}

$$\frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x + b^2*x^2],x]
```

```
[Out] (2*ArcTanh[(b*x)/Sqrt[b*x + b^2*x^2]])/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-b^2x^2}\,dx, x, \frac{x}{\sqrt{bx+b^2x^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{bx+b^2x^2}}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.
time = 0.04, size = 56, normalized size = 2.33

$$-\frac{2\sqrt{x}\,\sqrt{1+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{1+bx}\right)}{\sqrt{b}\,\sqrt{bx(1+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x + b^2*x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[1 + b*x]])/(Sqrt[b]
*Sqrt[b*x*(1 + b*x)])
```

Maple [A]
time = 0.37, size = 37, normalized size = 1.54

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2+bx}\right)}{\sqrt{b^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 29, normalized size = 1.21

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx}\,b+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x)*b + b)/b
```

Fricas [A]
time = 1.56, size = 27, normalized size = 1.12

$$-\frac{\log\left(-2\,bx+2\,\sqrt{b^2x^2+bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b^2*x^2 + b*x) - 1)/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^2 + bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b**2*x**2 + b*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.
time = 2.22, size = 59, normalized size = 2.46

$$\frac{1}{4}\sqrt{b^2x^2 + bx}\left(2\,x + \frac{1}{b}\right) + \frac{\log\left(\left|-2\left(x|b| - \sqrt{b^2x^2 + bx}\right)|b| - b\right|\right)}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b^2*x^2 + b*x)*(2*x + 1/b) + 1/8*log(abs(-2*(x*abs(b) - sqrt(b^2*x
^2 + b*x))*abs(b) - b))/abs(b)
```

Mupad [**B**]
time = 0.23, size = 36, normalized size = 1.50

$$\frac{\ln\left(\frac{x\,b^2 + \frac{b}{2}}{\sqrt{b^2}} + \sqrt{b^2\,x^2 + b\,x}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + b^2*x^2)^(1/2),x)
```

```
[Out] log((b/2 + b^2*x)/(b^2)^(1/2) + (b*x + b^2*x^2)^(1/2))/(b^2)^(1/2)
```

3.27 $\int \frac{1}{\sqrt{6x-x^2}}\,dx$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{3}\right)$$

```
[Out] arcsin(-1+1/3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {633, 222}

$$-\text{ArcSin}\left(1-\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[6*x - x^2],x]
```

```
[Out] -ArcSin[1 - x/3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{6x-x^2}}\,dx &= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 6-2x\right)\right)\\ &= -\sin^{-1}\left(1-\frac{x}{3}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(10) = 20.

time = 0.03, size = 38, normalized size = 3.80

$$\frac{2\sqrt{-6+x}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-6+x}}\right)}{\sqrt{-((-6+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[6*x - x^2],x]
```

```
[Out] (2*Sqrt[-6 + x]*Sqrt[x]*ArcTanh[Sqrt[x]/Sqrt[-6 + x]])/Sqrt[-((-6 + x)*x)]
```

Maple [A]
time = 0.38, size = 7, normalized size = 0.70

method	result	size
default	$\arcsin\left(-1+\frac{x}{3}\right)$	7
meijerg	$2\arcsin\left(\frac{\sqrt{6}\,\sqrt{x}}{6}\right)$	12
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-x\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2+6x}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(-1+1/3*x)
```

Maxima [A]
time = 0.50, size = 8, normalized size = 0.80

$$-\arcsin\left(-\frac{1}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/3*x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.
time = 1.66, size = 18, normalized size = 1.80

$$-2\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + 6*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 + 6x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+6*x)**(1/2),x)
[Out] Integral(1/sqrt(-x**2 + 6*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.
time = 2.84, size = 25, normalized size = 2.50

$$\frac{1}{2}\sqrt{-x^2 + 6\,x}\,(x-3) + \frac{9}{2}\arcsin\left(\frac{1}{3}\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

Mupad [B]
time = 0.11, size = 6, normalized size = 0.60

$$\mathrm{asin}\left(\frac{x}{3} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x - x^2)^(1/2),x)
[Out] asin(x/3 - 1)
```

3.28 $\int \frac{1}{\sqrt{4x+x^2}}\,dx$

Optimal. Leaf size=16

$$2\tanh^{-1}\left(\frac{x}{\sqrt{4x+x^2}}\right)$$

```
[Out] 2*arctanh(x/(x^2+4*x)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {634, 212}

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4*x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4x+x^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{4x+x^2}}\right)\\ &= 2\tanh^{-1}\left(\frac{x}{\sqrt{4x+x^2}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.
time = 0.03, size = 37, normalized size = 2.31

$$\frac{2\sqrt{x}\,\sqrt{4+x}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{4+x}}\right)}{\sqrt{x(4+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4*x + x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[4 + x]*ArcTanh[Sqrt[x]/Sqrt[4 + x]])/Sqrt[x*(4 + x)]
```

Maple [A]
time = 0.37, size = 14, normalized size = 0.88

method	result	size
meijerg	$2\,\mathrm{arcsinh}\left(\frac{\sqrt{x}}{2}\right)$	9
default	$\ln\left(2+x+\sqrt{x^2+4x}\right)$	14
trager	$\ln\left(2+x+\sqrt{x^2+4x}\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(2+x+(x^2+4*x)^(1/2))
```

Maxima [A]
time = 0.27, size = 17, normalized size = 1.06

$$\log\left(2\,x+2\,\sqrt{x^2+4\,x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 + 4*x) + 4)
```

Fricas [A]
time = 1.45, size = 17, normalized size = 1.06

$$-\log\left(-x+\sqrt{x^2+4\,x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 4*x) - 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^2+4x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + 4*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 1.97, size = 33, normalized size = 2.06

$$\frac{1}{2}\sqrt{x^2+4\,x}\,(x+2)+2\,\log\left(\left|-x+\sqrt{x^2+4\,x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

Mupad [B]
time = 0.51, size = 11, normalized size = 0.69

$$\ln\left(x+\sqrt{x\,(x+4)}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + x^2)^(1/2),x)
```

```
[Out] log(x + (x*(x + 4))^(1/2) + 2)
```

3.29 $\int \frac{1}{\sqrt{-2x+x^2}}\,dx$

Optimal. Leaf size=16

$$2\tanh^{-1}\left(\frac{x}{\sqrt{-2x+x^2}}\right)$$

```
[Out] 2*arctanh(x/(x^2-2*x)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {634, 212}

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2*x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[-2*x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2x+x^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-2x+x^2}}\right)\\ &= 2\tanh^{-1}\left(\frac{x}{\sqrt{-2x+x^2}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.
time = 0.03, size = 37, normalized size = 2.31

$$\frac{2\sqrt{-2+x}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2*x + x^2],x]
[Out] (2*Sqrt[-2 + x]*Sqrt[x]*ArcTanh[Sqrt[x]/Sqrt[-2 + x]])/Sqrt[(-2 + x)*x]
```

Maple [A]
time = 0.44, size = 14, normalized size = 0.88

method	result	size
default	$\ln\left(x-1+\sqrt{x^{2}-2x}\right)$	14
trager	$\ln\left(x-1+\sqrt{x^{2}-2x}\right)$	14
meijerg	$\frac{2\sqrt{-\mathrm{signum}\left(x-2\right)}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{\sqrt{\mathrm{signum}\left(x-2\right)}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x-1+(x^2-2*x)^(1/2))
```

Maxima [A]
time = 0.27, size = 17, normalized size = 1.06

$$\log\left(2\,x+2\,\sqrt{x^{2}-2\,x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 - 2*x) - 2)
```

Fricas [A]
time = 1.61, size = 17, normalized size = 1.06

$$-\log\left(-x+\sqrt{x^{2}-2\,x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 2*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^{2}-2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 - 2*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 1.95, size = 33, normalized size = 2.06

$$\frac{1}{2}\sqrt{x^2-2x}\,(x-1)+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2-2x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 - 2*x)*(x - 1) + 1/2*log(abs(-x + sqrt(x^2 - 2*x) + 1))
```

Mupad [B]
time = 0.53, size = 11, normalized size = 0.69

$$\ln\left(x+\sqrt{x\,(x-2)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x)^(1/2),x)
```

```
[Out] log(x + (x*(x - 2))^(1/2) - 1)
```

3.30 $\int (bx+cx^2)^{4/3}\,dx$

Optimal. Leaf size=448

$$\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(b+2cx)\,(bx+cx^2)^{4/3}}{55c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\,(bx+cx^2)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}+\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2(b}{}$$

```
[Out] 3/55*(-c*x*(c*x+b)/b^2)^(1/3)*(2*c*x+b)*(c*x^2+b*x)^(4/3)/c/(-c*(c*x^2+b*x)
/b^2)^(4/3)+3/22*(-c*x*(c*x+b)/b^2)^(4/3)*(2*c*x+b)*(c*x^2+b*x)^(4/3)/c/(-c
*(c*x^2+b*x)/b^2)^(4/3)+1/55*2^(1/3)*3^(3/4)*b^2*(c*x^2+b*x)^(4/3)*(1-2^(2/
3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+
3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6
^(1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c
*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c/(
2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(4/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/
(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {636, 633, 201, 242, 225}

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2(bx+cx^2)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{55c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\,(bx+cx^2)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}+\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(b+2cx)\,(bx+cx^2)^{4/3}}{55c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(4/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(4/3))/(55*c*(-
((c*(b*x + c*x^2))/b^2))^(4/3)) + (3*(-((c*x*(b + c*x))/b^2))^(4/3)*(b + 2*
c*x)*(b*x + c*x^2)^(4/3))/(22*c*(-((c*(b*x + c*x^2))/b^2))^(4/3)) + (2^(1/3
)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(b*x + c*x^2)^(4/3)*(1 - 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*
2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b +
```

```
 c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b +
 c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(55*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(4/3)*Sqrt[
-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c
*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \left(bx+cx^2\right)^{4/3}\,dx = \frac{\left(bx+cx^2\right)^{4/3}\int \left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{4/3}\,dx}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}$$

$$= -\frac{\left(b^2\left(bx+cx^2\right)^{4/3}\right)\text{Subst}\left(\int \left(1-\frac{b^2x^2}{c^2}\right)^{4/3}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{8\ 2^{2/3}c^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} - \frac{\left(b^2\left(bx+cx^2\right)^{4/3}\right)\text{Subst}\left(\int \sqrt[3]{1-\frac{b^2x^2}{c^2}}\,d\right.}{11\ 2^{2/3}c^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} -$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} +$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 48, normalized size = 0.11

$$\frac{3bx^2\sqrt[3]{x(b+cx)}\ {}_2F_1\left(-\frac{4}{3},\frac{7}{3};\frac{10}{3};-\frac{cx}{b}\right)}{7\sqrt[3]{1+\frac{cx}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(4/3),x]
```

```
[Out] (3*b*x^2*(x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3, 7/3, 10/3, -((c*x)/b)]
)/(7*(1 + (c*x)/b)^(1/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(c\,x^{2}+b x \right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(4/3),x)
```

```
[Out] int((c*x^2+b*x)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(4/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b x + c x^{2}\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(4/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(4/3), x)
```

Mupad [B]
time = 0.23, size = 36, normalized size = 0.08

$$\frac{3\,x\,\left(c\,x^2+b\,x\right)^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{c\,x}{b}\right)}{7\,\left(\frac{c\,x}{b}+1\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(4/3),x)
```

```
[Out] (3*x*(b*x + c*x^2)^(4/3)*hypergeom([-4/3, 7/3], 10/3, -(c*x)/b))/(7*((c*x)/
b + 1)^(4/3))
```

3.31 $\int \sqrt[3]{bx+cx^2}\,dx$

Optimal. Leaf size=387

$$\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}+\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\cdots}{\left(\cdots\right.}}\cdots}{5\;2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(b\cdots}{\cdots}}\cdots}$$

```
[Out] 3/10*(-c*x*(c*x+b)/b^2)^(1/3)*(2*c*x+b)*(c*x^2+b*x)^(1/3)/c/(-c*(c*x^2+b*x)
/b^2)^(1/3)+1/10*3^(3/4)*b^2*(c*x^2+b*x)^(1/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2
)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*
(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*
((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1
-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)*2^(1/3)/c/(2*c*x+b)/(-c
*(c*x^2+b*x)/b^2)^(1/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(
-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {636, 633, 201, 242, 225}

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{5\;2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(1/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(1/3))/(10*c*(-
((c*(b*x + c*x^2))/b^2))^(1/3)) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(b*x + c*x
^2)^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
```

```
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*2^(2/3)*c*(b + 2*c*x
)*(-((c*(b*x + c*x^2))/b^2))^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \sqrt[3]{bx+cx^2}\,dx = \frac{\sqrt[3]{bx+cx^2}\int \sqrt[3]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}\,dx}{\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}$$

$$= -\frac{\left(b^2\sqrt[3]{bx+cx^2}\right)\operatorname{Subst}\left(\int \sqrt[3]{1-\frac{b^2x^2}{c^2}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{2\ 2^{2/3}c^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} - \frac{\left(b^2\sqrt[3]{bx+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{2/3}}\,dx, x, -\frac{c}{b}\right.}{5\ 2^{2/3}c^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{\left(3\sqrt[3]{bx+cx^2}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\operatorname{Subst}\left(\right.}{10\ 2^{2/3}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}$$

$$= \frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\sqrt[3]{bx+cx^2}\sqrt{-1-\frac{4cx}{b}-\frac{4c}{}}}{5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.39, size = 45, normalized size = 0.12

$$\frac{3x\sqrt[3]{x(b+cx)}\ {}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};-\frac{cx}{b}\right)}{4\sqrt[3]{1+\frac{cx}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(1/3),x]
```

```
[Out] (3*x*(x*(b + c*x))^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -((c*x)/b)])/(4*
(1 + (c*x)/b)^(1/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(c\,x^{2}+b x \right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/3),x)
```

```
[Out] int((c*x^2+b*x)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b x + c x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(1/3), x)
```

Mupad [**B**]
time = 0.17, size = 36, normalized size = 0.09

$$\frac{3\,x\,{\left(c\,x^2+b\,x\right)}^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{4}{3};\ \frac{7}{3};\ -\frac{c\,x}{b}\right)}{4\,{\left(\frac{c\,x}{b}+1\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(1/3),x)
```

```
[Out] (3*x*(b*x + c*x^2)^(1/3)*hypergeom([-1/3, 4/3], 7/3, -(c*x)/b))/(4*((c*x)/b
 + 1)^(1/3))
```

3.32 $\int \frac{1}{\left(bx+cx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=322

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+2\sqrt[3]{2}\left(-\frac{cx}{\cdots}\right.}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right.}}}{c(b+2cx)\left(bx+cx^2\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+\cdots}{b^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(\cdots}{\cdots}}\right.}}}$$

```
[Out] 2^(1/3)*3^(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(2/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2
)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*
(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*
((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1
-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c/(2*c*x+b)/(c*x^2+b*x)
^(2/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)
^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {636, 633, 242, 225}

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{c(b+2cx)\left(bx+cx^2\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-2/3),x]
```

```
[Out] (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(2/3)*(1
- 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b +
```

```
 c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(2/3)*S
qrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(
-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{2/3}}\,dx = \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{2/3}}\,dx}{(bx+cx^2)^{2/3}}$$

$$= -\frac{\left(\sqrt[3]{2}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{2/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\,(bx+cx^2)^{2/3}}$$

$$= \frac{\left(3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, 2^{2/3}\sqrt[3]{-\frac{cx\,(1}{}}\right.}{2^{2/3}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)(bx+cx^2)^{2/3}}$$

$$= \frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right.}{c(b+2cx)\,(bx+cx^2)^{2/3}\sqrt{-1}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.28, size = 43, normalized size = 0.13

$$\frac{3x\left(1+\frac{cx}{b}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{cx}{b}\right)}{(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-2/3),x]
```

```
[Out] (3*x*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((c*x)/b)])/(x*(
b + c*x))^(2/3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\,x^2+bx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(2/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(2/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-2/3), x)
```

Mupad [B]
time = 0.22, size = 36, normalized size = 0.11

$$\frac{3\,x\,{\left(\frac{c\,x}{b}+1\right)}^{2/3}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{4}{3};\ -\frac{c\,x}{b}\right)}{{\left(c\,x^2+b\,x\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(2/3),x)
```

```
[Out] (3*x*((c*x)/b + 1)^(2/3)*hypergeom([1/3, 2/3], 4/3, -(c*x)/b))/(b*x + c*x^2
)^(2/3)
```

3.33 $\int \frac{1}{(bx+cx^2)^{5/3}}\,dx$

Optimal. Leaf size=384

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}}+\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{1+2^{2/3}\cdots}{(\cdots}}\cdots}{c(b+2cx)\,(bx+cx^2)^{5}\cdots}$$

[Out] 3/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(5/3)/c/(-c*x*(c*x+b)/b^2)^(2/3)/(c*x^2+b*x)^(5/3)+2^(1/3)*3^(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(5/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c/(2*c*x+b)/(c*x^2+b*x)^(5/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A]

time = 0.35, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {636, 633, 205, 242, 225}

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{c(b+2cx)\,(bx+cx^2)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-5/3),x]

[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3))/(2*c*(-((c*x*(b + c*x))/b^2))^(2/3)*(b*x + c*x^2)^(5/3)) + (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(5/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]

```
*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*
(b + 2*c*x)*(b*x + c*x^2)^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2)
)^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{5/3}}\,dx = \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{5/3}}\,dx}{(bx+cx^2)^{5/3}}$$

$$= -\frac{\left(4\sqrt[3]{2}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{5/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\,(bx+cx^2)^{5/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}} - \frac{\left(\sqrt[3]{2}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{2/3}}\,dx, x,\right.}{c^2\,(bx+cx^2)^{5/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}} + \frac{\left(3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\right.}{2^{2/3}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)(bx}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}} + \frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\sqrt{-1-\frac{4cx}{b}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 47, normalized size = 0.12

$$-\frac{3\left(1+\frac{cx}{b}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{1}{3};-\frac{cx}{b}\right)}{2b(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-5/3),x]
```

```
[Out] (-3*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 1/3, -((c*x)/b)])/(2*b
*(x*(b + c*x))^(2/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{5}{3}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(5/3),x)
[Out] int(1/(c*x^2+b*x)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x)^(-5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x)^(1/3)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b x + c x^{2}\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(5/3),x)
[Out] Integral((b*x + c*x**2)**(-5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/3), x)
```

Mupad [B]
time = 0.25, size = 36, normalized size = 0.09

$$-\frac{3\,x\,{\left(\frac{c\,x}{b}+1\right)}^{5/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{5}{3};\ \frac{1}{3};\ -\frac{c\,x}{b}\right)}{2\,{\left(c\,x^2+b\,x\right)}^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(5/3),x)
```

```
[Out] -(3*x*((c*x)/b + 1)^(5/3)*hypergeom([-2/3, 5/3], 1/3, -(c*x)/b))/(2*(b*x +
c*x^2)^(5/3))
```

3.34 $\int \frac{1}{(bx+cx^2)^{8/3}}\,dx$

Optimal. Leaf size=448

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}}+\frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}}+\frac{14\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\left(\cdots\right.}{\cdots}$$

```
[Out] 3/5*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(8/3)/c/(-c*x*(c*x+b)/b^2)^(5/3)/(c*x^2+
b*x)^(8/3)+21/5*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(8/3)/c/(-c*x*(c*x+b)/b^2)^(
2/3)/(c*x^2+b*x)^(8/3)+14/5*2^(1/3)*3^(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(8/3)*
(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2
)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2)
)*(1/2*6^(1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*
(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(
1/2)/c/(2*c*x+b)/(c*x^2+b*x)^(8/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(
1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {636, 633, 205, 242, 225}

$$\frac{14\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{5c(b+2cx)(bx+cx^2)^{8/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}}+\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-8/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(8/3))/(5*c*(-((c*x*(b + c*x))/b^
2))^(5/3)*(b*x + c*x^2)^(8/3)) + (21*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))
^(8/3))/(5*c*(-((c*x*(b + c*x))/b^2))^(2/3)*(b*x + c*x^2)^(8/3)) + (14*2^(1
/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(8/3)*(1 - 2^(
2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b
```

```
^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*c*(b + 2*c*x)*(b*x + c*x^2)^(8/3)*Sqrt
[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{8/3}}\,dx = \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{8/3}}\,dx}{(bx+cx^2)^{8/3}}$$

$$= -\frac{\left(16\sqrt[3]{2}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{8/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\,(bx+cx^2)^{8/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}} - \frac{\left(56\sqrt[3]{2}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{5/3}}\,d\cdots\right.}{5c^2\,(bx+cx^2)^{8/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}} + \frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}} - \frac{\left(14\sqrt[3]{2}\,b^2\left(-\frac{c(b\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}} + \frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}} + \frac{\left(21\sqrt[3]{2}\left(-\frac{c(bx+\cdots}{b^2}\right.\right.}{\cdots}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}} + \frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}} + \frac{14\sqrt[3]{2}\,3^{3/4}\sqrt{2-\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 50, normalized size = 0.11

$$-\frac{3\left(1+\frac{cx}{b}\right)^{2/3}\,{}_2F_1\left(-\frac{5}{3},\frac{8}{3};-\frac{2}{3};-\frac{cx}{b}\right)}{5b^2x(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-8/3),x]
```

```
[Out] (-3*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[-5/3, 8/3, -2/3, -((c*x)/b)])/(5*
b^2*x*(x*(b + c*x))^(2/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^{2}+bx\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(8/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-8/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(1/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx+cx^{2}\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(8/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-8/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-8/3), x)
```

Mupad [B]
time = 0.27, size = 36, normalized size = 0.08

$$-\frac{3\,x\,{\left(\frac{c\,x}{b}+1\right)}^{8/3}\,{}_2\mathrm{F}_1\left(-\frac{5}{3},\frac{8}{3};\ -\frac{2}{3};\ -\frac{c\,x}{b}\right)}{5\,{\left(c\,x^2+b\,x\right)}^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(8/3),x)
```

```
[Out] -(3*x*((c*x)/b + 1)^(8/3)*hypergeom([-5/3, 8/3], -2/3, -(c*x)/b))/(5*(b*x +
 c*x^2)^(8/3))
```

3.35 $\int (bx + cx^2)^{5/3} \, dx$

Optimal. Leaf size=842

$$\frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}-\frac{15(b+}{182\sqrt[3]{2}\,c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}$$

```
[Out] 15/364*(-c*x*(c*x+b)/b^2)^(2/3)*(2*c*x+b)*(c*x^2+b*x)^(5/3)/c/(-c*(c*x^2+b*
x)/b^2)^(5/3)+3/26*(-c*x*(c*x+b)/b^2)^(5/3)*(2*c*x+b)*(c*x^2+b*x)^(5/3)/c/(
-c*(c*x^2+b*x)/b^2)^(5/3)-15/364*(2*c*x+b)*(c*x^2+b*x)^(5/3)*2^(2/3)/c/(-c*
(c*x^2+b*x)/b^2)^(5/3)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))+5/182*3
^(3/4)*b^2*(c*x^2+b*x)^(5/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF
((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)
^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/
3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2
)^(1/2)*2^(1/6)/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(5/3)/((-1+2^(2/3)*(-c*x*(
c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)-15
/728*3^(1/4)*b^2*(c*x^2+b*x)^(5/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*Ell
ipticE((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b
)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2
*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1
/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(-c*(c*x^2+b*x)
/b^2)^(5/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)
/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.80, antiderivative size = 842, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {636, 633, 201, 241, 310, 225, 1893}

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(cx^2+bx\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)b^2}{364\sqrt[3]{2}\,c(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{5\,3^{3/4}\left(cx^2+bx\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)b^2}{91\,2^{5/6}c(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}-\frac{15(b+2cx)\left(cx^2+bx\right)^{5/3}}{182\sqrt[3]{2}\,c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(cx^2+bx\right)^{5/3}}{26c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}}+\frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(cx^2+bx\right)^{5/3}}{364c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/3),x]
```

```
[Out] (15*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(5/3))/(364*c*
(-((c*(b*x + c*x^2))/b^2))^(5/3)) + (3*(-((c*x*(b + c*x))/b^2))^(5/3)*(b +
2*c*x)*(b*x + c*x^2)^(5/3))/(26*c*(-((c*(b*x + c*x^2))/b^2))^(5/3)) - (15*(
b + 2*c*x)*(b*x + c*x^2)^(5/3))/(182*2^(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(
5/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (15*3^(1/4)*
Sqrt[2 + Sqrt[3]]*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-
((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*S
qrt[3]])/(364*2^(1/3)*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3)*Sqrt[-
((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*
x*(b + c*x))/b^2))^(1/3))^2)]) + (5*3^(3/4)*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^
(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/
b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2
/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2
/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(91*2^(5/6)*c*(b + 2*c*x)*(-((c*(b*x + c
*x^2))/b^2))^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 -
 Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
```

```
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{5/3} dx = \frac{\left(bx+cx^2\right)^{5/3} \int \left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{5/3} dx}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}$$

$$= -\frac{\left(b^2\left(bx+cx^2\right)^{5/3}\right) \text{Subst}\left(\int \left(1-\frac{b^2x^2}{c^2}\right)^{5/3} dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{16\sqrt[3]{2}\, c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} - \frac{\left(5b^2\left(bx+cx^2\right)^{5/3}\right) \text{Subst}\left(\int \left(1-\frac{b^2x^2}{c^2}\right)^{2/3}\right.}{104\sqrt[3]{2}\, c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/}}$$

$$= \frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} -$$

$$= \frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} +$$

$$= \frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} -$$

$$= \frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{364c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b+2cx)\left(bx+cx^2\right)^{5/3}}{26c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 10.03, size = 48, normalized size = 0.06

$$\frac{3bx^2(x(b+cx))^{2/3}\,{}_2F_1\left(-\frac{5}{3},\frac{8}{3};\frac{11}{3};-\frac{cx}{b}\right)}{8\left(1+\frac{cx}{b}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/3),x]
```

```
[Out] (3*b*x^2*(x*(b + c*x))^(2/3)*Hypergeometric2F1[-5/3, 8/3, 11/3, -((c*x)/b)]
)/(8*(1 + (c*x)/b)^(2/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+bx\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/3),x)
```

```
[Out] int((c*x^2+b*x)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(5/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx+cx^2\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/3), x)
```

Mupad [B]
time = 0.18, size = 36, normalized size = 0.04

$$\frac{3\,x\,{\left(c\,x^2+b\,x\right)}^{5/3}\,{}_2\mathrm{F}_1\left(-\frac{5}{3},\frac{8}{3};\ \frac{11}{3};\ -\frac{c\,x}{b}\right)}{8\,{\left(\frac{c\,x}{b}+1\right)}^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(5/3),x)
```

```
[Out] (3*x*(b*x + c*x^2)^(5/3)*hypergeom([-5/3, 8/3], 11/3, -(c*x)/b))/(8*((c*x)/
b + 1)^(5/3))
```

3.36 $\int \left(bx+cx^2\right)^{2/3}\,dx$

Optimal. Leaf size=781

$$\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}}-\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{7\sqrt[3]{2}\,c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}-\frac{3\sqrt[4]{3}\,\sqrt{\cdots}}{\cdots}$$

```
[Out] 3/14*(-c*x*(c*x+b)/b^2)^(2/3)*(2*c*x+b)*(c*x^2+b*x)^(2/3)/c/(-c*(c*x^2+b*x)
/b^2)^(2/3)-3/14*(2*c*x+b)*(c*x^2+b*x)^(2/3)*2^(2/3)/c/(-c*(c*x^2+b*x)/b^2)
^(2/3)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))+1/7*2^(1/6)*3^(3/4)*b^2
*(c*x^2+b*x)^(2/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3
)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(
1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(
c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c/
(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(2/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))
/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)-3/28*3^(1/4)*b^2*(c*
x^2+b*x)^(2/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticE((1-2^(2/3)*(-
c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+
b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(2/3)/((-1+2^(
2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))
^2)^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {636, 633, 201, 241, 310, 225, 1893}

$$\frac{\sqrt[6]{2}\,3^{3/4}b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{14\sqrt[3]{2}\,c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}}-\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{7\sqrt[3]{2}\,c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(2/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(2/3))/(14*c*(-
((c*(b*x + c*x^2))/b^2))^(2/3)) - (3*(b + 2*c*x)*(b*x + c*x^2)^(2/3))/(7*2^
(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(2/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b
+ c*x))/b^2))^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(b*x + c*x^2)^(2/3
)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[
3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[
3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(14*2^(1/3)*c*(b + 2*c*x)*(-((c
*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1
/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)
*3^(3/4)*b^2*(b*x + c*x^2)^(2/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b
+ c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))
^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))
/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/
(7*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[-((1 - 2^(2/3)*(-((c
*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(bx+cx^2\right)^{2/3}\,dx = \frac{\left(bx+cx^2\right)^{2/3}\int\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{2/3}\,dx}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}}$$

$$= -\frac{\left(b^2\left(bx+cx^2\right)^{2/3}\right)\text{Subst}\left(\int\left(1-\frac{b^2x^2}{c^2}\right)^{2/3}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{4\sqrt[3]{2}\,c^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} - \frac{\left(b^2\left(bx+cx^2\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{1-\frac{b^2x^2}{c^2}}}\,d\right.}{7\sqrt[3]{2}\,c^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} + \frac{\left(3\left(bx+cx^2\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{S}}{14\sqrt[3]{2}\left(-\frac{c}{b}-\right.}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} - \frac{\left(3\left(bx+cx^2\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{S}}{14\sqrt[3]{2}\left(-\frac{c}{b}-\right.}$$

$$= \frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} + \frac{3b^2\left(bx+cx^2\right)^{2/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}}{7\sqrt[3]{2}\,c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(1-\sqrt{3}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 45, normalized size = 0.06

$$\frac{3x(x(b+cx))^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};-\frac{cx}{b}\right)}{5\left(1+\frac{cx}{b}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(2/3),x]
```

```
[Out] (3*x*(x*(b + c*x))^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((c*x)/b)])/(5*
(1 + (c*x)/b)^(2/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(c\,x^{2}+b x \right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(2/3),x)
```

```
[Out] int((c*x^2+b*x)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b x + c x^{2}\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(2/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(2/3), x)
```

Mupad [B]
time = 0.16, size = 36, normalized size = 0.05

$$\frac{3\,x\,{\left(c\,x^2+b\,x\right)}^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{5}{3};\ \frac{8}{3};\ -\frac{c\,x}{b}\right)}{5\,{\left(\frac{c\,x}{b}+1\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(2/3),x)
```

```
[Out] (3*x*(b*x + c*x^2)^(2/3)*hypergeom([-2/3, 5/3], 8/3, -(c*x)/b))/(5*((c*x)/b
 + 1)^(2/3))
```

3.37 $\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx$

Optimal. Leaf size=715

$$-\frac{3(b+2cx)\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}}{\sqrt[3]{2}\,c\sqrt[3]{bx+cx^2}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}-\frac{3\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{c\ldots}{\ldots}}\right.}{2\ldots}$$

```
[Out] -3/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(1/3)*2^(2/3)/c/(c*x^2+b*x)^(1/3)/(1-2^
(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))+2^(1/6)*3^(3/4)*b^2*(-c*(c*x^2+b*x)
/b^2)^(1/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x
*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2
*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/
b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c/(2*c*x+
b)/(c*x^2+b*x)^(1/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*
x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)-3/4*3^(1/4)*b^2*(-c*(c*x^2+b*x)/b^2)
^(1/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticE((1-2^(2/3)*(-c*x*(c*x
+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*
3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^
(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1
/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(c*x^2+b*x)^(1/3)/((-1+2^(2/3)*(-c*x*(c*x+b
)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.60, antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {636, 633, 241, 310, 225, 1893}

$$\frac{\sqrt[6]{2}\,3^{3/4}b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}-\frac{3\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\sqrt[3]{2}\,c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}-\frac{3(b+2cx)\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}}{\sqrt[3]{2}\,c\sqrt[3]{bx+cx^2}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-1/3),x]
```

```
[Out] (-3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(1/3))/(2^(1/3)*c*(b*x + c*x^2)^
(1/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (3*3^(1/4)*
```

```
Sqrt[2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(1/3)*(1 - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) +
 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(
b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(
b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))], -7 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(1/3)*Sqrt[-((
1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*
(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)*3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^
2))^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c
*x^2)^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3
] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
```

```
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx = \frac{\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[3]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}\,dx}{\sqrt[3]{bx+cx^2}}$$

$$= -\frac{\left(b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\frac{b^2x^2}{c^2}}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{\sqrt[3]{2}\,c^2\sqrt[3]{bx+cx^2}}$$

$$= \frac{\left(3\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, 2^{2/3}\sqrt[3]{-\frac{cx\,(1}{}}\right.}{2\sqrt[3]{2}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\sqrt[3]{bx+cx^2}}$$

$$= -\frac{\left(3\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, 2^{2/3}\sqrt[3]{-\frac{cx}{}}\right.}{2\sqrt[3]{2}\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\sqrt[3]{bx+cx^2}}$$

$$= \frac{3b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\sqrt{-1-\frac{4cx(b+cx)}{b^2}}}{\sqrt[3]{2}\,c(b+2cx)\sqrt[3]{bx+cx^2}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,b^{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.59, size = 45, normalized size = 0.06

$$\frac{3x\sqrt[3]{1+\frac{cx}{b}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{cx}{b}\right)}{2\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-1/3),x]
```

```
[Out] (3*x*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((c*x)/b)])/(2*(
x*(b + c*x))^(1/3))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^{2}+bx\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(1/3),x)
[Out] int(1/(c*x^2+b*x)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x)^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x)^(-1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(1/3),x)
[Out] Integral((b*x + c*x**2)**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x)^(-1/3), x)
```

Mupad [B]
time = 0.20, size = 36, normalized size = 0.05

$$\frac{3\,x\,{\left(\frac{c\,x}{b}+1\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{c\,x}{b}\right)}{2\,{\left(c\,x^2+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(1/3),x)
[Out] (3*x*((c*x)/b + 1)^(1/3)*hypergeom([1/3, 2/3], 5/3, -(c*x)/b))/(2*(b*x + c*
x^2)^(1/3))
```

3.38 $\int \frac{1}{\left(bx+cx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=773

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{4/3}}+\frac{3\ 2^{2/3}(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\left(bx+cx^2\right)^{4/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,b^2\Big(-\frac{c}{\cdots}}{\cdots}$$

```
[Out] 3*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(4/3)/c/(-c*x*(c*x+b)/b^2)^(1/3)/(c*x^2+b*
x)^(4/3)+3*2^(2/3)*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(4/3)/c/(c*x^2+b*x)^(4/3)
/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))-2*2^(1/6)*3^(3/4)*b^2*(-c*(c*
x^2+b*x)/b^2)^(4/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/
3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^
(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*
(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)/c
/(2*c*x+b)/(c*x^2+b*x)^(4/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2
/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)+3/2*3^(1/4)*b^2*(-c*(c*x^2+b
*x)/b^2)^(4/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticE((1-2^(2/3)*(-
c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+
b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(c*x^2+b*x)^(4/3)/((-1+2^(2/3)*(-c*
x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 773, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {636, 633, 205, 241, 310, 225, 1893}

$$-\frac{2\sqrt[6]{2}\,3^{3/4}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{c(b+2cx)\left(bx+cx^2\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{2}\,c(b+2cx)\left(bx+cx^2\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(bx+cx^2\right)^{4/3}}+\frac{3\ 2^{2/3}(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\left(bx+cx^2\right)^{4/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-4/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(4/3))/(c*(-((c*x*(b + c*x))/b^2)
)^(1/3)*(b*x + c*x^2)^(4/3)) + (3*2^(2/3)*(b + 2*c*x)*(-((c*(b*x + c*x^2))/
```

```
b^2))^(4/3))/(c*(b*x + c*x^2)^(4/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^
2))^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(2^(1/3)*c*(b + 2*c*x)*
(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1
- Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]) - (2*2^(1/6)*3^(3/
4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((
c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqr
t[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b +
c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2
)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{4/3}}\,dx = \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{4/3}}\,dx}{(bx+cx^2)^{4/3}}$$

$$= -\frac{\left(2\ 2^{2/3}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{4/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\,(bx+cx^2)^{4/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{4/3}} + \frac{\left(2^{2/3}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\frac{b^2x^2}{c^2}}}\,dx \cdots\right.}{c^2\,(bx+cx^2)^{4/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{4/3}} - \frac{\left(3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\cdots}{\sqrt[3]{2}\,\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\cdots}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{4/3}} + \frac{\left(3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\right)\text{Subst}\cdots}{\sqrt[3]{2}\,\left(-\frac{c}{b}-\frac{2c^2x}{b^2}\right)\cdots}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{4/3}} - \frac{3\ 2^{2/3}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\sqrt{-1-\frac{4cx}{b}-\frac{4c^2x^2}{b^2}}\sqrt{-\cdots}}{c(b+2cx)\,(bx+cx^2)^{4/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 45, normalized size = 0.06

$$-\frac{3\sqrt[3]{1+\frac{cx}{b}}\ {}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{2}{3};-\frac{cx}{b}\right)}{b\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-4/3),x]
[Out] (-3*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, -((c*x)/b)])/(b*(
x*(b + c*x))^(1/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^{2}+bx\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(4/3),x)
[Out] int(1/(c*x^2+b*x)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x)^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x)^(2/3)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx+cx^{2}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(4/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-4/3), x)
```

Mupad [B]
time = 0.23, size = 36, normalized size = 0.05

$$-\frac{3\,x\,{\left(\frac{c\,x}{b}+1\right)}^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{4}{3};\ \frac{2}{3};\ -\frac{c\,x}{b}\right)}{{\left(c\,x^2+b\,x\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(4/3),x)
```

```
[Out] -(3*x*((c*x)/b + 1)^(4/3)*hypergeom([-1/3, 4/3], 2/3, -(c*x)/b))/(b*x + c*x
^2)^(4/3)
```

3.39 $\int \frac{1}{(bx+cx^2)^{7/3}}\,dx$

Optimal. Leaf size=838

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(bx+cx^2)^{7/3}}+\frac{15(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{7/3}}+\frac{15(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{\sqrt[3]{2}\,c\,(bx+cx^2)^{7/3}\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{c}{\cdots}}\right.}$$

```
[Out] 3/4*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(7/3)/c/(-c*x*(c*x+b)/b^2)^(4/3)/(c*x^2+
b*x)^(7/3)+15/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(7/3)/c/(-c*x*(c*x+b)/b^2)^(
1/3)/(c*x^2+b*x)^(7/3)+15/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(7/3)*2^(2/3)/c/
(c*x^2+b*x)^(7/3)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))-5*2^(1/6)*3^
(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(7/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*E
llipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x
+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)
+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^
(1/2))^2)^(1/2)/c/(2*c*x+b)/(c*x^2+b*x)^(7/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^
2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2)^(1/2)+15/4*3^(1/4
)*b^2*(-c*(c*x^2+b*x)/b^2)^(7/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*Ellip
ticE((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/
b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2
^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2
))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(c*x^2+b*x)^(7/3)
/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)
-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 1.12, antiderivative size = 838, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {636, 633, 205, 241, 310, 225, 1893}

$$\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{7/3}}{2\sqrt[3]{2}\,c(b+2cx)\left(cx^2+bx\right)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}-\frac{5\sqrt[6]{2}\,3^{3/4}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{7/3}}{c(b+2cx)\left(cx^2+bx\right)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{15(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{7/3}}{\sqrt[3]{2}\,c\left(cx^2+bx\right)^{7/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}+\frac{15(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(cx^2+bx\right)^{7/3}}+\frac{3(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}\left(cx^2+bx\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-7/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(7/3))/(4*c*(-((c*x*(b + c*x))/b^
2))^(4/3)*(b*x + c*x^2)^(7/3)) + (15*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))
^(7/3))/(2*c*(-((c*x*(b + c*x))/b^2))^(1/3)*(b*x + c*x^2)^(7/3)) + (15*(b +
 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(7/3))/(2^(1/3)*c*(b*x + c*x^2)^(7/3)*(1
 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) + (15*3^(1/4)*Sqrt[2
+ Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(7/3)*(1 - 2^(2/3)*(-((c*x*(b + c
*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1
/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7
 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(7/3)*Sqrt[-((1 - 2^(
2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3))^2)]) - (5*2^(1/6)*3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(
7/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)
^(7/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2
^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
```

```
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{7/3}}\,dx = \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{7/3}}\,dx}{(bx+cx^2)^{7/3}}$$

$$= -\frac{\left(8\ 2^{2/3}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{7/3}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\,(bx+cx^2)^{7/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(bx+cx^2)^{7/3}} - \frac{\left(5\ 2^{2/3}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{4/3}}\,dx\right.}{c^2\,(bx+cx^2)^{7/3}}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(bx+cx^2)^{7/3}} + \frac{15(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{7/3}} + \frac{\left(5b^2\left(-\frac{c(bx+cx^2}{b^2}\right.\right.}{}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(bx+cx^2)^{7/3}} + \frac{15(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{7/3}} - \frac{\left(15\left(-\frac{c(bx+cx^2)}{b^2}\right.\right.}{}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(bx+cx^2)^{7/3}} + \frac{15(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{7/3}} + \frac{\left(15\left(-\frac{c(bx+cx^2)}{b^2}\right.\right.}{}$$

$$= \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(bx+cx^2)^{7/3}} + \frac{15(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{7/3}} - \frac{15b^2\left(-\frac{c(bx+cx^2)}{b^2}\right.}{\sqrt[3]{2}\,c(b+2cx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 50, normalized size = 0.06

$$-\frac{3\sqrt[3]{1+\frac{cx}{b}}\ {}_2F_1\left(-\frac{4}{3},\frac{7}{3};-\frac{1}{3};-\frac{cx}{b}\right)}{4b^2x\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-7/3),x]
```

```
[Out] (-3*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[-4/3, 7/3, -1/3, -((c*x)/b)])/(4*
b^2*x*(x*(b + c*x))^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^{2}+bx\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(7/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(7/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-7/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(2/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3
), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+cx^2)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(7/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-7/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-7/3), x)
```

Mupad [**B**]
time = 0.25, size = 36, normalized size = 0.04

$$-\frac{3\,x\,{\left(\frac{c\,x}{b}+1\right)}^{7/3}\,{{}}_2{\mathrm{F}}_1\left(-\frac{4}{3},\frac{7}{3};\ -\frac{1}{3};\ -\frac{c\,x}{b}\right)}{4\,{\left(c\,x^2+b\,x\right)}^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(7/3),x)
```

```
[Out] -(3*x*((c*x)/b + 1)^(7/3)*hypergeom([-4/3, 7/3], -1/3, -(c*x)/b))/(4*(b*x +
 c*x^2)^(7/3))
```

3.40 $\int \left(bx+cx^2\right)^{5/4}\,dx$

Optimal. Leaf size=119

$$-\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}+\frac{5b^5\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{84\sqrt{2}\,c^3\left(bx+cx^2\right)^{3/4}}$$

```
[Out] -5/84*b^2*(2*c*x+b)*(c*x^2+b*x)^(1/4)/c^2+1/7*(2*c*x+b)*(c*x^2+b*x)^(5/4)/c
+5/168*b^5*(-c*(c*x^2+b*x)/b^2)^(3/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/
cos(1/2*arcsin(1+2*c*x/b))*EllipticF(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))/c^
3/(c*x^2+b*x)^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {626, 636, 633, 238}

$$\frac{5b^5\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{84\sqrt{2}\,c^3\left(bx+cx^2\right)^{3/4}}-\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(5/4),x]
```

```
[Out] (-5*b^2*(b + 2*c*x)*(b*x + c*x^2)^(1/4))/(84*c^2) + ((b + 2*c*x)*(b*x + c*x
^2)^(5/4))/(7*c) + (5*b^5*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin
[1 + (2*c*x)/b]/2, 2])/(84*Sqrt[2]*c^3*(b*x + c*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int (bx+cx^2)^{5/4}\,dx = \frac{(b+2cx)(bx+cx^2)^{5/4}}{7c} - \frac{(5b^2)\int \sqrt[4]{bx+cx^2}\,dx}{28c}$$
$$= -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{(b+2cx)(bx+cx^2)^{5/4}}{7c} + \frac{(5b^4)\int \frac{1}{(bx+cx^2)^{3/4}}\,dx}{336c^2}$$
$$= -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{(b+2cx)(bx+cx^2)^{5/4}}{7c} + \frac{\left(5b^4\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right)\int \frac{}{\left(-\frac{cx}{b}\right.}}{336c^2(bx+cx^2)^{3/}}$$
$$= -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{(b+2cx)(bx+cx^2)^{5/4}}{7c} - \frac{\left(5b^6\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right)\text{Subst}}{168\sqrt{2}\,c^4}$$
$$= -\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{(b+2cx)(bx+cx^2)^{5/4}}{7c} + \frac{5b^5\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\right.}{84\sqrt{2}\,c^3(bx+cx^2)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 48, normalized size = 0.40

$$\frac{4bx^2\sqrt[4]{x(b+cx)}\;{}_2F_1\left(-\frac{5}{4},\frac{9}{4};\frac{13}{4};-\frac{cx}{b}\right)}{9\sqrt[4]{1+\frac{cx}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(5/4),x]
[Out] (4*b*x^2*(x*(b + c*x))^(1/4)*Hypergeometric2F1[-5/4, 9/4, 13/4, -((c*x)/b)]
)/(9*(1 + (c*x)/b)^(1/4))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+bx\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(5/4),x)
```

```
[Out] int((c*x^2+b*x)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(5/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx+cx^2\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(5/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/4), x)
```

Mupad [B]
time = 0.19, size = 36, normalized size = 0.30

$$\frac{4\,x\,{\left(c\,x^2+b\,x\right)}^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{9}{4};\ \frac{13}{4};\ -\frac{c\,x}{b}\right)}{9\,{\left(\frac{c\,x}{b}+1\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(5/4),x)
```

```
[Out] (4*x*(b*x + c*x^2)^(5/4)*hypergeom([-5/4, 9/4], 13/4, -(c*x)/b))/(9*((c*x)/
b + 1)^(5/4))
```

3.41 $\int (bx+cx^2)^{3/4}\,dx$

Optimal. Leaf size=90

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{10\sqrt{2}\,c^2\sqrt[4]{bx+cx^2}}$$

```
[Out] 1/5*(2*c*x+b)*(c*x^2+b*x)^(3/4)/c-3/20*b^3*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(
1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(1/
2*arcsin(1+2*c*x/b)),2^(1/2))/c^2/(c*x^2+b*x)^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {626, 636, 633, 234}

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{10\sqrt{2}\,c^2\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(3/4),x]
```

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(3/4))/(5*c) - (3*b^3*(-((c*(b*x + c*x^2))/b^2))
^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(10*Sqrt[2]*c^2*(b*x + c*x^2)
^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{(3b^2)\int\frac{1}{\sqrt[4]{bx+cx^2}}\,dx}{20c}$$

$$=\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{\left(3b^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\int\frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}\,dx}{20c\sqrt[4]{bx+cx^2}}$$

$$=\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}+\frac{\left(3b^4\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{1-\frac{b^2x^2}{c^2}}}\,dx,x,-\frac{c}{b}-\right.}{20\sqrt{2}\,c^3\sqrt[4]{bx+cx^2}}$$

$$=\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^3\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{10\sqrt{2}\,c^2\sqrt[4]{bx+cx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 45, normalized size = 0.50

$$\frac{4x(x(b+cx))^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{11}{4};-\frac{cx}{b}\right)}{7\left(1+\frac{cx}{b}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(3/4),x]
```

```
[Out] (4*x*(x*(b + c*x))^(3/4)*Hypergeometric2F1[-3/4, 7/4, 11/4, -((c*x)/b)])/(7
*(1 + (c*x)/b)^(3/4))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+bx\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(3/4),x)
```

```
[Out] int((c*x^2+b*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx + cx^2\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(3/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(3/4), x)
```

Mupad [B]
time = 0.17, size = 36, normalized size = 0.40

$$\frac{4\,x\,{\left(c\,x^2+b\,x\right)}^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{7}{4};\ \frac{11}{4};\ -\frac{c\,x}{b}\right)}{7\,{\left(\frac{c\,x}{b}+1\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(3/4),x)
[Out] (4*x*(b*x + c*x^2)^(3/4)*hypergeom([-3/4, 7/4], 11/4, -(c*x)/b))/(7*((c*x)/
b + 1)^(3/4))
```

3.42 $\int \sqrt[4]{bx+cx^2}\,dx$

Optimal. Leaf size=90

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{3\sqrt{2}\,c^2\left(bx+cx^2\right)^{3/4}}$$

```
[Out] 1/3*(2*c*x+b)*(c*x^2+b*x)^(1/4)/c-1/6*b^3*(-c*(c*x^2+b*x)/b^2)^(3/4)*(cos(1
/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticF(sin(1/2
*arcsin(1+2*c*x/b)),2^(1/2))/c^2/(c*x^2+b*x)^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {626, 636, 633, 238}

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{3\sqrt{2}\,c^2\left(bx+cx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(1/4),x]
```

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(1/4))/(3*c) - (b^3*(-((c*(b*x + c*x^2))/b^2))^(
3/4)*EllipticF[ArcSin[1 + (2*c*x)/b]/2, 2])/(3*Sqrt[2]*c^2*(b*x + c*x^2)^(3
/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\begin{aligned}\int \sqrt[4]{bx+cx^2}\,dx &= \frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\int \frac{1}{(bx+cx^2)^{3/4}}\,dx}{12c}\\ &= \frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{\left(b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right)\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{3/4}}\,dx}{12c\,(bx+cx^2)^{3/4}}\\ &= \frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}+\frac{\left(b^4\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{3/4}}\,dx,x,-\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{6\sqrt{2}\,c^3\,(bx+cx^2)^{3/4}}\\ &= \frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|\,2\right)}{3\sqrt{2}\,c^2\,(bx+cx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 45, normalized size = 0.50

$$\frac{4x\sqrt[4]{x(b+cx)}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{9}{4};-\frac{cx}{b}\right)}{5\sqrt[4]{1+\frac{cx}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(1/4),x]
```

```
[Out] (4*x*(x*(b + c*x))^(1/4)*Hypergeometric2F1[-1/4, 5/4, 9/4, -((c*x)/b)])/(5*
(1 + (c*x)/b)^(1/4))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+bx\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^(1/4),x)
[Out] int((c*x^2+b*x)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/4),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/4),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/4),x)
[Out] Integral((b*x + c*x**2)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^(1/4),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x)^(1/4), x)
```

Mupad [B]
time = 0.17, size = 36, normalized size = 0.40

$$\frac{4\,x\,{\left(c\,x^2+b\,x\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{c\,x}{b}\right)}{5\,{\left(\frac{c\,x}{b}+1\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(1/4),x)
```

```
[Out] (4*x*(b*x + c*x^2)^(1/4)*hypergeom([-1/4, 5/4], 9/4, -(c*x)/b))/(5*((c*x)/b
 + 1)^(1/4))
```

3.43 $\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{2}\,b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|\,2\right)}{c\sqrt[4]{bx+cx^2}}$$

```
[Out] b*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*a
rcsin(1+2*c*x/b))*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/2)/c/(
c*x^2+b*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {636, 633, 234}

$$\frac{\sqrt{2}\,b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{2cx}{b}+1\right)\middle|\,2\right)}{c\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-1/4),x]
```

```
[Out] (Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]
/2, 2])/(c*(b*x + c*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \frac{\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}\,dx}{\sqrt[4]{bx+cx^2}}$$

$$= -\frac{\left(b^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{b^2x^2}{c^2}}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{\sqrt{2}\,c^2\sqrt[4]{bx+cx^2}}$$

$$= \frac{\sqrt{2}\,b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|\,2\right)}{c\sqrt[4]{bx+cx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 45, normalized size = 0.78

$$\frac{4x\sqrt[4]{1+\frac{cx}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{cx}{b}\right)}{3\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-1/4),x]
```

```
[Out] (4*x*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((c*x)/b)])/(3*(
x*(b + c*x))^(1/4))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^2+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(1/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x)^(-1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x)^(-1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(1/4),x)
[Out] Integral((b*x + c*x**2)**(-1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x)^(-1/4), x)
```

Mupad [B]
time = 0.20, size = 36, normalized size = 0.62

$$\frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{c\,x}{b}\right)}{3\,\left(c\,x^2+b\,x\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(1/4),x)
[Out] (4*x*((c*x)/b + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -(c*x)/b))/(3*(b*x + c*
x^2)^(1/4))
```

3.44 $\int \frac{1}{(bx+cx^2)^{3/4}} \, dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{2}\, b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{c\left(bx+cx^2\right)^{3/4}}$$

```
[Out] 2*b*(-c*(c*x^2+b*x)/b^2)^(3/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2
*arcsin(1+2*c*x/b))*EllipticF(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/2)/c
/(c*x^2+b*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {636, 633, 238}

$$\frac{2\sqrt{2}\, b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\text{ArcSin}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c\left(bx+cx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[1 + (2*c*x)/
b]/2, 2])/(c*(b*x + c*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\int \frac{1}{\left(-\frac{cx}{b}-\frac{c^2x^2}{b^2}\right)^{3/4}}\,dx}{(bx+cx^2)^{3/4}}$$

$$= -\frac{\left(\sqrt{2}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{b^2x^2}{c^2}\right)^{3/4}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right)}{c^2\,(bx+cx^2)^{3/4}}$$

$$= \frac{2\sqrt{2}\,b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|\,2\right)}{c\,(bx+cx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 43, normalized size = 0.73

$$\frac{4x\left(1+\frac{cx}{b}\right)^{3/4} {}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{cx}{b}\right)}{(x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-3/4),x]
[Out] (4*x*(1 + (c*x)/b)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((c*x)/b)])/(x*(
b + c*x))^(3/4)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\,x^2+bx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(3/4),x)
[Out] int(1/(c*x^2+b*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(3/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-3/4), x)
```

Mupad [B]
time = 0.19, size = 36, normalized size = 0.61

$$\frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ -\frac{c\,x}{b}\right)}{\left(c\,x^2+b\,x\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(3/4),x)
```

```
[Out] (4*x*((c*x)/b + 1)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(c*x)/b))/(b*x + c*x^2
)^(3/4)
```

3.45 $\int \frac{1}{(bx+cx^2)^{5/4}}\,dx$

Optimal. Leaf size=83

$$-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}+\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}$$

```
[Out] -4*(2*c*x+b)/b^2/(c*x^2+b*x)^(1/4)+4*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*ar
csin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(1/2*arcs
in(1+2*c*x/b)),2^(1/2))*2^(1/2)/b/(c*x^2+b*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {628, 636, 633, 234}

$$\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-5/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4)) + (4*Sqrt[2]*(-((c*(b*x + c*x^2)
)/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(b*(b*x + c*x^2)^(1/4)
)
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}} + \frac{(4c)\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx}{b^2}$$

$$= -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}} + \frac{\left(4c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}\,dx}{b^2\sqrt[4]{bx+cx^2}}$$

$$= -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}} - \frac{\left(2\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{b^2x^2}{c^2}}}\,dx, x, -\frac{c}{b}-\frac{2c^2x}{b^2}\right.}{c\sqrt[4]{bx+cx^2}}$$

$$= -\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}} + \frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|\,2\right)}{b\sqrt[4]{bx+cx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 45, normalized size = 0.54

$$-\frac{4\sqrt[4]{1+\frac{cx}{b}}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};-\frac{cx}{b}\right)}{b\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-5/4),x]
[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((c*x)/b)])/(b*(
x*(b + c*x))^(1/4))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(5/4),x)
[Out] int(1/(c*x^2+b*x)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x)^(-5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x)^(3/4)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx + cx^{2}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(5/4),x)
[Out] Integral((b*x + c*x**2)**(-5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/4), x)
```

Mupad [B]
time = 0.22, size = 36, normalized size = 0.43

$$-\frac{4\,x\,{\left(\frac{c\,x}{b}+1\right)}^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ -\frac{c\,x}{b}\right)}{{\left(c\,x^2+b\,x\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(5/4),x)
```

```
[Out] -(4*x*((c*x)/b + 1)^(5/4)*hypergeom([-1/4, 5/4], 3/4, -(c*x)/b))/(b*x + c*x
^2)^(5/4)
```

3.46 $\int \frac{1}{(bx+cx^2)^{9/4}}\,dx$

Optimal. Leaf size=115

$$-\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}}+\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}}-\frac{48\sqrt{2}\,c\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|\,2\right)}{5b^3\sqrt[4]{bx+cx^2}}$$

```
[Out] -4/5*(2*c*x+b)/b^2/(c*x^2+b*x)^(5/4)+48/5*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(1/4)
-48/5*c*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos
(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/
2)/b^3/(c*x^2+b*x)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$,
Rules used = {628, 636, 633, 234}

$$-\frac{48\sqrt{2}\,c\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{2cx}{b}+1\right)\middle|\,2\right)}{5b^3\sqrt[4]{bx+cx^2}}+\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-9/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/4)) + (48*c*(b + 2*c*x))/(5*b^4*(b
*x + c*x^2)^(1/4)) - (48*Sqrt[2]*c*(-((c*(b*x + c*x^2))/b^2))^(1/4)*Ellipti
cE[ArcSin[1 + (2*c*x)/b]/2, 2])/(5*b^3*(b*x + c*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = -\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}} - \frac{(12c)\int \frac{1}{(bx+cx^2)^{5/4}}\,dx}{5b^2}$$

$$= -\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}} + \frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{(48c^2)\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx}{5b^4}$$

$$= -\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}} + \frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{\left(48c^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}}$$

$$5b^4\sqrt[4]{bx+cx^2}$$

$$= -\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}} + \frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} + \frac{\left(24\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-}}\right.}{5b^2\sqrt[4]{bx+cx^2}}$$

$$= -\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}} + \frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{48\sqrt{2}\,c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\right.}{5b^3\sqrt[4]{bx+cx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 50, normalized size = 0.43

$$-\frac{4\sqrt[4]{1+\frac{cx}{b}}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};-\frac{1}{4};-\frac{cx}{b}\right)}{5b^2x\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-9/4),x]
```

```
[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, -((c*x)/b)])/(5*
b^2*x*(x*(b + c*x))^(1/4))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(9/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(9/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-9/4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/4)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3
), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b x + c x^{2}\right)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(9/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-9/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-9/4), x)
```

Mupad [B]
time = 0.26, size = 36, normalized size = 0.31

$$-\frac{4\,x\,{\left(\frac{c\,x}{b}+1\right)}^{9/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{9}{4};\ -\frac{1}{4};\ -\frac{c\,x}{b}\right)}{5\,{\left(c\,x^2+b\,x\right)}^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(9/4),x)
```

```
[Out] -(4*x*((c*x)/b + 1)^(9/4)*hypergeom([-5/4, 9/4], -1/4, -(c*x)/b))/(5*(b*x +
 c*x^2)^(9/4))
```

3.47 $\int \frac{1}{(bx+cx^2)^{13/4}}\,dx$

Optimal. Leaf size=146

$$-\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}}+\frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}}-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}}+\frac{448\sqrt{2}\,c^2\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\,E\left(\frac{1}{2}\sin^{-1}\left(1+\frac{2cx}{b}\right)\middle|2\right)}{15b^5\sqrt[4]{bx+cx^2}}$$

```
[Out] -4/9*(2*c*x+b)/b^2/(c*x^2+b*x)^(9/4)+112/45*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(5/
4)-448/15*c^2*(2*c*x+b)/b^6/(c*x^2+b*x)^(1/4)+448/15*c^2*(-c*(c*x^2+b*x)/b^
2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*El
lipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/2)/b^5/(c*x^2+b*x)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {628, 636, 633, 234}

$$\frac{448\sqrt{2}\,c^2\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{15b^5\sqrt[4]{bx+cx^2}}-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}}+\frac{112c(b+2cx)}{45b^4\left(bx+cx^2\right)^{5/4}}-\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^(-13/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(9*b^2*(b*x + c*x^2)^(9/4)) + (112*c*(b + 2*c*x))/(45*b^4*
(b*x + c*x^2)^(5/4)) - (448*c^2*(b + 2*c*x))/(15*b^6*(b*x + c*x^2)^(1/4)) +
 (448*Sqrt[2]*c^2*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*
c*x)/b]/2, 2])/(15*b^5*(b*x + c*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = -\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}} - \frac{(28c)\int \frac{1}{(bx+cx^2)^{9/4}}\,dx}{9b^2}$$

$$= -\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}} + \frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}} + \frac{(112c^2)\int \frac{1}{(bx+cx^2)^{5/4}}\,dx}{15b^4}$$

$$= -\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}} + \frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}} - \frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} + \frac{(448c^3)\int \frac{1}{\sqrt[4]{bx+cx^2}} \cdots}{15b^6}$$

$$= -\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}} + \frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}} - \frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} + \frac{\left(448c^3\sqrt[4]{-\frac{c(bx+cx \cdots}{b^2}} \cdots\right.}{15b^6 \cdots}$$

$$= -\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}} + \frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}} - \frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} - \frac{\left(224\sqrt{2}\,c\sqrt[4]{-\frac{c(bx+ \cdots}{b^2}} \cdots\right.}{\cdots}$$

$$= -\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}} + \frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}} - \frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} + \frac{448\sqrt{2}\,c^2\sqrt[4]{-\frac{c(bx+ \cdots}{b^2}} \cdots}{15b^5 \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 50, normalized size = 0.34

$$-\frac{4\sqrt[4]{1+\frac{cx}{b}}\,{}_2F_1\left(-\frac{9}{4},\frac{13}{4};-\frac{5}{4};-\frac{cx}{b}\right)}{9b^3x^2\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-13/4),x]
```

```
[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-9/4, 13/4, -5/4, -((c*x)/b)])/(9
*b^3*x^2*(x*(b + c*x))^(1/4))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^{2}+b x \right)^{\frac{13}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x)^(13/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(13/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-13/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(3/4)/(c^4*x^8 + 4*b*c^3*x^7 + 6*b^2*c^2*x^6 + 4*b^3
*c*x^5 + b^4*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b x + c x^{2}\right)^{\frac{13}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(13/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-13/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-13/4), x)
```

Mupad [B]
time = 0.29, size = 36, normalized size = 0.25

$$-\frac{4\,x\,{\left(\frac{c\,x}{b}+1\right)}^{13/4}\,{{}}_2{\mathrm{F}}_1\left(-\frac{9}{4},\frac{13}{4};\ -\frac{5}{4};\ -\frac{c\,x}{b}\right)}{9\,{\left(c\,x^2+b\,x\right)}^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2)^(13/4),x)
```

```
[Out] -(4*x*((c*x)/b + 1)^(13/4)*hypergeom([-9/4, 13/4], -5/4, -(c*x)/b))/(9*(b*x
 + c*x^2)^(13/4))
```

3.48 $\int \left(bx+cx^2\right)^p \, dx$

Optimal. Leaf size=55

$$-\frac{\left(-\frac{cx}{b}\right)^{-1-p}\left(bx+cx^2\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+cx}{b}\right)}{b(1+p)}$$

```
[Out] -(-c*x/b)^(-1-p)*(c*x^2+b*x)^(1+p)*hypergeom([-p, 1+p],[2+p],(c*x+b)/b)/b/(
1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {638}

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1}\left(bx+cx^2\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{b+cx}{b}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)^p,x]
```

```
[Out] -(((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p,
 2 + p, (b + c*x)/b])/(b*(1 + p)))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int \left(bx+cx^2\right)^p \, dx=-\frac{\left(-\frac{cx}{b}\right)^{-1-p}\left(bx+cx^2\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+cx}{b}\right)}{b(1+p)}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.82

$$\frac{x(x(b+cx))^p\left(1+\frac{cx}{b}\right)^{-p}\,{}_2F_1\left(-p,1+p;2+p;-\frac{cx}{b}\right)}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^p,x]
```

```
[Out] (x*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x)/b)])/((1 + p
)*(1 + (c*x)/b)^p)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(c\,x^{2}+b x \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)^p,x)
```

```
[Out] int((c*x^2+b*x)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b x + c x^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**p,x)
```

```
[Out] Integral((b*x + c*x**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^p, x)
```

Mupad [B]
time = 0.32, size = 48, normalized size = 0.87

$$\frac{x\,\left(c\,x^2+b\,x\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ -\frac{c\,x}{b}\right)}{{\left(\frac{c\,x}{b}+1\right)}^p\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^p,x)
```

```
[Out] (x*(b*x + c*x^2)^p*hypergeom([-p, p + 1], p + 2, -(c*x)/b))/(((c*x)/b + 1)^
p*(p + 1))
```

3.49 $\int (a+cx^2)^4 \, dx$

Optimal. Leaf size=51

$$a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

```
[Out] a^4*x+4/3*a^3*c*x^3+6/5*a^2*c^2*x^5+4/7*a*c^3*x^7+1/9*c^4*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {200}

$$a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^4,x]
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^4 \, dx &= \int \left(a^4+4a^3cx^2+6a^2c^2x^4+4ac^3x^6+c^4x^8\right) \, dx\\ &= a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 51, normalized size = 1.00

$$a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^4,x]
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9
```

Maple [A]
time = 0.41, size = 44, normalized size = 0.86

method	result	size
gosper	$a^4x + \frac{4}{3}c\,a^3x^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}a\,c^3x^7 + \frac{1}{9}c^4x^9$	44
default	$a^4x + \frac{4}{3}c\,a^3x^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}a\,c^3x^7 + \frac{1}{9}c^4x^9$	44
norman	$a^4x + \frac{4}{3}c\,a^3x^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}a\,c^3x^7 + \frac{1}{9}c^4x^9$	44
risch	$a^4x + \frac{4}{3}c\,a^3x^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}a\,c^3x^7 + \frac{1}{9}c^4x^9$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^4,x,method=_RETURNVERBOSE)
[Out] a^4*x+4/3*c*a^3*x^3+6/5*a^2*c^2*x^5+4/7*a*c^3*x^7+1/9*c^4*x^9
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.84

$$\frac{1}{9}\,c^4x^9 + \frac{4}{7}\,ac^3x^7 + \frac{6}{5}\,a^2c^2x^5 + \frac{4}{3}\,a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="maxima")
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

Fricas [A]
time = 1.70, size = 43, normalized size = 0.84

$$\frac{1}{9}\,c^4x^9 + \frac{4}{7}\,ac^3x^7 + \frac{6}{5}\,a^2c^2x^5 + \frac{4}{3}\,a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="fricas")
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.96

$$a^4x + \frac{4a^3cx^3}{3} + \frac{6a^2c^2x^5}{5} + \frac{4ac^3x^7}{7} + \frac{c^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**4,x)
```

```
[Out] a**4*x + 4*a**3*c*x**3/3 + 6*a**2*c**2*x**5/5 + 4*a*c**3*x**7/7 + c**4*x**9
/9
```

Giac [**A**]
time = 0.99, size = 43, normalized size = 0.84

$$\frac{1}{9}\,c^4x^9 + \frac{4}{7}\,ac^3x^7 + \frac{6}{5}\,a^2c^2x^5 + \frac{4}{3}\,a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x
```

Mupad [**B**]
time = 0.03, size = 43, normalized size = 0.84

$$a^4\,x + \frac{4\,a^3\,c\,x^3}{3} + \frac{6\,a^2\,c^2\,x^5}{5} + \frac{4\,a\,c^3\,x^7}{7} + \frac{c^4\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^4,x)
```

```
[Out] a^4*x + (c^4*x^9)/9 + (4*a^3*c*x^3)/3 + (4*a*c^3*x^7)/7 + (6*a^2*c^2*x^5)/5
```

3.50 $\int (a+cx^2)^3 \, dx$

Optimal. Leaf size=35

$$a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

[Out] a^3*x+a^2*c*x^3+3/5*a*c^2*x^5+1/7*c^3*x^7

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {200}

$$a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^3,x]

[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int (a+cx^2)^3 \, dx &= \int \left(a^3 + 3a^2cx^2 + 3ac^2x^4 + c^3x^6\right) dx \\ &= a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^3,x]

[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7

Maple [A]
time = 0.36, size = 32, normalized size = 0.91

method	result	size
gosper	$a^3x+a^2c\,x^3+\frac{3}{5}a\,c^2x^5+\frac{1}{7}c^3x^7$	32
default	$a^3x+a^2c\,x^3+\frac{3}{5}a\,c^2x^5+\frac{1}{7}c^3x^7$	32
norman	$a^3x+a^2c\,x^3+\frac{3}{5}a\,c^2x^5+\frac{1}{7}c^3x^7$	32
risch	$a^3x+a^2c\,x^3+\frac{3}{5}a\,c^2x^5+\frac{1}{7}c^3x^7$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+a^2*c*x^3+3/5*a*c^2*x^5+1/7*c^3*x^7
```

Maxima [A]
time = 0.32, size = 31, normalized size = 0.89

$$\frac{1}{7}\,c^3x^7+\frac{3}{5}\,ac^2x^5+a^2cx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

Fricas [A]
time = 1.48, size = 31, normalized size = 0.89

$$\frac{1}{7}\,c^3x^7+\frac{3}{5}\,ac^2x^5+a^2cx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

Sympy [A]
time = 0.01, size = 32, normalized size = 0.91

$$a^3x+a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7
```

Giac [**A**]
time = 1.00, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

Mupad [**B**]
time = 0.04, size = 31, normalized size = 0.89

$$a^3\,x+a^2\,c\,x^3+\frac{3\,a\,c^2\,x^5}{5}+\frac{c^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3,x)
```

```
[Out] a^3*x + (c^3*x^7)/7 + a^2*c*x^3 + (3*a*c^2*x^5)/5
```

3.51 $\int (a+cx^2)^2 \, dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*c*x^3+1/5*c^2*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{200\}$

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(a+cx^2\right)^2 \, dx &= \int \left(a^2+2acx^2+c^2x^4\right) \, dx \\ &= a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Maple [A]
time = 0.36, size = 22, normalized size = 0.88

method	result	size
gosper	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}c^2x^5$	22
default	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}c^2x^5$	22
norman	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}c^2x^5$	22
risch	$a^2x+\frac{2}{3}ac\,x^3+\frac{1}{5}c^2x^5$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/3*a*c*x^3+1/5*c^2*x^5
```

Maxima [A]
time = 0.26, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,acx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

Fricas [A]
time = 1.49, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,acx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.88

$$a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*c*x**3/3 + c**2*x**5/5
```

Giac [A]
time = 0.87, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,c\,x^3}{3} + \frac{c^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2,x)
```

```
[Out] a^2*x + (c^2*x^5)/5 + (2*a*c*x^3)/3
```

3.52 $\int (a + cx^2) \, dx$

Optimal. Leaf size=12

$$ax + \frac{cx^3}{3}$$

```
[Out] a*x+1/3*c*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + c*x^2,x]
```

```
[Out] a*x + (c*x^3)/3
```

Rubi steps

$$\int (a + cx^2) \, dx = ax + \frac{cx^3}{3}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + c*x^2,x]
```

```
[Out] a*x + (c*x^3)/3
```

Maple [A]
time = 0.02, size = 11, normalized size = 0.92

method	result	size
gosper	$ax + \frac{1}{3}c\,x^3$	11
default	$ax + \frac{1}{3}c\,x^3$	11

norman	$ax+\frac{1}{3}c\,x^3$	11
risch	$ax+\frac{1}{3}c\,x^3$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2+a,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/3*c*x^3
```

Maxima [A]
time = 0.29, size = 10, normalized size = 0.83

$$\frac{1}{3}\,cx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + a*x
```

Fricas [A]
time = 1.57, size = 10, normalized size = 0.83

$$\frac{1}{3}\,cx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3 + a*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$ax+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2+a,x)
```

```
[Out] a*x + c*x**3/3
```

Giac [A]
time = 1.35, size = 10, normalized size = 0.83

$$\frac{1}{3}\,cx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2+a,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + a*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{c\,x^3}{3}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + c*x^2,x)
```

```
[Out] a*x + (c*x^3)/3
```

3.53 $\int \frac{1}{a+cx^2} \, dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

```
[Out] arctan(x*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+cx^2} \, dx = \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Maple [A]
time = 0.40, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$	16
risch	$-\frac{\ln\left(cx+\sqrt{-ac}\right)}{2\sqrt{-ac}}+\frac{\ln\left(-cx+\sqrt{-ac}\right)}{2\sqrt{-ac}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Maxima [A]
time = 0.52, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="maxima")
[Out] arctan(c*x/sqrt(a*c))/sqrt(a*c)
```

Fricas [A]
time = 1.75, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ac}\,\log\left(\frac{cx^2-2\sqrt{-ac}\,x-a}{cx^2+a}\right)}{2\,ac},\ \frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))/(a*c), sqrt(
a*c)*arctan(sqrt(a*c)*x/a)/(a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

time = 0.05, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ac}}\,\log\left(-a\sqrt{-\frac{1}{ac}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ac}}\,\log\left(a\sqrt{-\frac{1}{ac}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*c))*log(-a*sqrt(-1/(a*c)) + x)/2 + sqrt(-1/(a*c))*log(a*sqrt(-1
/(a*c)) + x)/2
```

Giac [A]
time = 1.44, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(a*c))/sqrt(a*c)
```

Mupad [B]
time = 0.07, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2),x)
```

```
[Out] atan((c^(1/2)*x)/a^(1/2))/(a^(1/2)*c^(1/2))
```

3.54 $\int \frac{1}{(a+cx^2)^2} \, dx$

Optimal. Leaf size=45

$$\frac{x}{2a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

```
[Out] 1/2*x/a/(c*x^2+a)+1/2*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+cx^2\right)^2}\,dx &= \frac{x}{2a\left(a+cx^2\right)}+\frac{\int \frac{1}{a+cx^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x}{2a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-2),x]
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
```

Maple [A]
time = 0.39, size = 36, normalized size = 0.80

method	result	size
default	$\frac{x}{2a(c\,x^2+a)}+\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2a\sqrt{ac}}$	36
risch	$\frac{x}{2a(c\,x^2+a)}-\frac{\ln\left(cx+\sqrt{-ac}\right)}{4\sqrt{-ac}\;a}+\frac{\ln\left(-cx+\sqrt{-ac}\right)}{4\sqrt{-ac}\;a}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x/a/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 35, normalized size = 0.78

$$\frac{x}{2\left(acx^2+a^2\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*x/(a*c*x^2 + a^2) + 1/2*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a)
```

Fricas [A]
time = 1.51, size = 120, normalized size = 2.67

$$\left[\frac{2\,acx-\left(cx^2+a\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)}{4\left(a^2c^2x^2+a^3c\right)},\frac{acx+\left(cx^2+a\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)}{2\left(a^2c^2x^2+a^3c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*x - (c*x^2 + a)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*
x^2 + a)))/(a^2*c^2*x^2 + a^3*c), 1/2*(a*c*x + (c*x^2 + a)*sqrt(a*c)*arctan
(sqrt(a*c)*x/a))/(a^2*c^2*x^2 + a^3*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.09, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2acx^2} - \frac{\sqrt{-\frac{1}{a^3c}}\log\left(-a^2\sqrt{-\frac{1}{a^3c}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c}}\log\left(a^2\sqrt{-\frac{1}{a^3c}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*c*x**2) - sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c)) + x
)/4 + sqrt(-1/(a**3*c))*log(a**2*sqrt(-1/(a**3*c)) + x)/4
```

Giac [A]
time = 1.00, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,a} + \frac{x}{2\,(cx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*x/((c*x^2 + a)*a)
```

Mupad [B]
time = 0.14, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(c\,x^2 + a\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^2,x)
```

```
[Out] x/(2*a*(a + c*x^2)) + atan((c^(1/2)*x)/a^(1/2))/(2*a^(3/2)*c^(1/2))
```

3.55 $\int \frac{1}{(a+cx^2)^3}\,dx$

Optimal. Leaf size=62

$$\frac{x}{4a\left(a+cx^2\right)^2}+\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

```
[Out] 1/4*x/a/(c*x^2+a)^2+3/8*x/a^2/(c*x^2+a)+3/8*arctan(x*c^(1/2)/a^(1/2))/a^(5/
2)/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{x}{4a\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a + c*x^2)^2) + (3*x)/(8*a^2*(a + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+cx^2)^3}\,dx = \frac{x}{4a\,(a+cx^2)^2} + \frac{3\int \frac{1}{(a+cx^2)^2}\,dx}{4a}$$
$$= \frac{x}{4a\,(a+cx^2)^2} + \frac{3x}{8a^2\,(a+cx^2)} + \frac{3\int \frac{1}{a+cx^2}\,dx}{8a^2}$$
$$= \frac{x}{4a\,(a+cx^2)^2} + \frac{3x}{8a^2\,(a+cx^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.89

$$\frac{5ax+3cx^3}{8a^2\,(a+cx^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-3),x]
[Out] (5*a*x + 3*c*x^3)/(8*a^2*(a + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(
8*a^(5/2)*Sqrt[c])
```

Maple [A]
time = 0.39, size = 57, normalized size = 0.92

method	result	size
default	$\frac{x}{4a(c\,x^2+a)^2}+\frac{\frac{3x}{8a\left(c\,x^2+a\right)}+\frac{3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a\sqrt{ac}}}{a}$	57
risch	$\frac{\frac{3c\,x^3}{8a^2}+\frac{5x}{8a}}{(c\,x^2+a)^2}-\frac{3\ln\left(cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\;a^2}+\frac{3\ln\left(-cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\;a^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*x/a/(c*x^2+a)^2+3/4/a*(1/2*x/a/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(c*x/(
a*c)^(1/2)))
```

Maxima [A]
time = 0.52, size = 58, normalized size = 0.94

$$\frac{3\,cx^3+5\,ax}{8\,(a^2c^2x^4+2\,a^3cx^2+a^4)} + \frac{3\,\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c*x^3 + 5*a*x)/(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4) + 3/8*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^2)
```

Fricas [A]
time = 1.91, size = 188, normalized size = 3.03

$$\left[\frac{6\,ac^2x^3+10\,a^2cx-3\,(c^2x^4+2\,acx^2+a^2)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)}{16\,(a^3c^3x^4+2\,a^4c^2x^2+a^5c)},\ \frac{3\,ac^2x^3+5\,a^2cx+3\,(c^2x^4+2\,acx^2+a^2)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)}{8\,(a^3c^3x^4+2\,a^4c^2x^2+a^5c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*c^2*x^3 + 10*a^2*c*x - 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(-a*c)*
log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2
 + a^5*c), 1/8*(3*a*c^2*x^3 + 5*a^2*c*x + 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqr
t(a*c)*arctan(sqrt(a*c)*x/a))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c)]
```

Sympy [A]
time = 0.15, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{5ax+3cx^3}{8a^4+16a^3cx^2+8a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt(-1/(a**5*
c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16 + (5*a*x + 3*c*x**3)/(8*a**4 + 16*a*
*3*c*x**2 + 8*a**2*c**2*x**4)
```

Giac [A]
time = 1.61, size = 45, normalized size = 0.73

$$\frac{3\,\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2}+\frac{3\,cx^3+5\,ax}{8\,(cx^2+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/8*(3*c*x^3 + 5*a*x)/((c*x^2 +
 a)^2*a^2)
```

Mupad [B]
time = 0.16, size = 55, normalized size = 0.89

$$\frac{\frac{5\,x}{8\,a}+\frac{3\,c\,x^3}{8\,a^2}}{a^2+2\,a\,c\,x^2+c^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^3,x)
[Out] ((5*x)/(8*a) + (3*c*x^3)/(8*a^2))/(a^2 + c^2*x^4 + 2*a*c*x^2) + (3*atan((c^
(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2))
```

3.56 $\int (a+cx^2)^{5/2}\,dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}$$

```
[Out] 5/24*a*x*(c*x^2+a)^(3/2)+1/6*x*(c*x^2+a)^(5/2)+5/16*a^3*arctanh(x*c^(1/2)/(
c*x^2+a)^(1/2))/c^(1/2)+5/16*a^2*x*(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + c*x^2])/16 + (5*a*x*(a + c*x^2)^(3/2))/24 + (x*(a + c*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^{5/2} dx &= \frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{1}{6}(5a)\int \left(a+cx^2\right)^{3/2} dx\\ &= \frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \sqrt{a+cx^2}\, dx\\ &= \frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+cx^2}}dx\\ &= \frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-cx^2}d\right.\\ &= \frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 0.85

$$\frac{1}{48}\sqrt{a+cx^2}\left(33a^2x+26acx^3+8c^2x^5\right)-\frac{5a^3\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{16\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(33*a^2*x + 26*a*c*x^3 + 8*c^2*x^5))/48 - (5*a^3*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2]])/(16*Sqrt[c])
```

Maple [A]
time = 0.38, size = 68, normalized size = 0.81

method	result	size
risch	$\frac{x\left(8c^2x^4+26c\,x^2a+33a^2\right)\sqrt{c\,x^2+a}}{48}+\frac{5a^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{16\sqrt{c}}$	59
default	$\frac{x\left(c\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{6}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(c*x^2+a)^(5/2)+5/6*a*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(
1/2)+1/2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.69

$$\frac{1}{6}\left(cx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(cx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{cx^2+a}\,a^2x+\frac{5\,a^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(5/2)*x + 5/24*(c*x^2 + a)^(3/2)*a*x + 5/16*sqrt(c*x^2 + a)
*a^2*x + 5/16*a^3*arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

Fricas [A]
time = 1.36, size = 146, normalized size = 1.74

$$\left[\frac{15\,a^3\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,(8\,c^3x^5+26\,ac^2x^3+33\,a^2cx)\sqrt{cx^2+a}}{96\,c},-\frac{15\,a^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-(8\,c^3x^5+26\,ac^2x^3+33\,a^2cx)\sqrt{cx^2+a}}{48\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(
8*c^3*x^5 + 26*a*c^2*x^3 + 33*a^2*c*x)*sqrt(c*x^2 + a))/c, -1/48*(15*a^3*sq
rt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (8*c^3*x^5 + 26*a*c^2*x^3 + 33*
a^2*c*x)*sqrt(c*x^2 + a))/c]
```

Sympy [A]
time = 2.56, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}cx^3\sqrt{1+\frac{cx^2}{a}}}{24}+\frac{\sqrt{a}\,c^2x^5\sqrt{1+\frac{cx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + c*x**2/a)/16 + 13*a**(3/2)*c*x**3*sqrt(1 + c*x**2/a)
/24 + sqrt(a)*c**2*x**5*sqrt(1 + c*x**2/a)/6 + 5*a**3*asinh(sqrt(c)*x/sqrt(
a))/(16*sqrt(c))
```

Giac [A]
time = 1.51, size = 63, normalized size = 0.75

$$-\frac{5\,a^3\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{16\,\sqrt{c}}+\frac{1}{48}\left(2\left(4\,c^2x^2+13\,ac\right)x^2+33\,a^2\right)\sqrt{cx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/48*(2*(4*c^2*x
^2 + 13*a*c)*x^2 + 33*a^2)*sqrt(c*x^2 + a)*x
```

Mupad [B]
time = 0.20, size = 37, normalized size = 0.44

$$\frac{x\,{\left(c\,x^2+a\right)}^{5/2}\,{{}}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{c\,x^2}{a}\right)}{{\left(\frac{c\,x^2}{a}+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(5/2),x)
```

```
[Out] (x*(a + c*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(c*x^2)/a))/((c*x^2)/a +
1)^(5/2)
```

3.57 $\int \left(a+cx^2\right)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}$$

```
[Out] 1/4*x*(c*x^2+a)^(3/2)+3/8*a^2*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(1/2)+3/
8*a*x*(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + c*x^2])/8 + (x*(a + c*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+cx^2\right)^{3/2}\,dx = \frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+cx^2}\,dx$$
$$= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)$$
$$= \frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}$$

Mathematica [A]
time = 0.06, size = 60, normalized size = 0.92

$$\frac{1}{8}x\sqrt{a+cx^2}\left(5a+2cx^2\right)-\frac{3a^2\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{8\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[a + c*x^2]*(5*a + 2*c*x^2))/8 - (3*a^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Maple [A]
time = 0.38, size = 52, normalized size = 0.80

method	result	size
risch	$\frac{x(2c\,x^2+5a)\sqrt{c\,x^2+a}}{8}+\frac{3a^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8\sqrt{c}}$	48
default	$\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.30, size = 43, normalized size = 0.66

$$\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{cx^2+a}\,ax+\frac{3\,a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2 + a)^(3/2)*x + 3/8*sqrt(c*x^2 + a)*a*x + 3/8*a^2*arcsinh(c*x/sqr
t(a*c))/sqrt(c)
```

Fricas [A]
time = 1.59, size = 124, normalized size = 1.91

$$\left[\frac{3\,a^2\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,(2\,c^2x^3+5\,acx)\sqrt{cx^2+a}}{16\,c},-\frac{3\,a^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-(2\,c^2x^3+5\,acx)\sqrt{cx^2+a}}{8\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2
*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c, -1/8*(3*a^2*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (2*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c]
```

Sympy [A]
time = 1.56, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{cx^2}{a}}}{8}+\frac{\sqrt{a}\,cx^3\sqrt{1+\frac{cx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + c*x**2/a)/8 + sqrt(a)*c*x**3*sqrt(1 + c*x**2/a)/4 + 3
*a**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c))
```

Giac [A]
time = 1.21, size = 49, normalized size = 0.75

$$\frac{1}{8}\left(2\,cx^2+5\,a\right)\sqrt{cx^2+a}\,x-\frac{3\,a^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*c*x^2 + 5*a)*sqrt(c*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(c)*x + sqrt(c
*x^2 + a)))/sqrt(c)
```

Mupad [B]
time = 0.16, size = 37, normalized size = 0.57

$$\frac{x\,{\left(c\,x^2+a\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{c\,x^2}{a}\right)}{{\left(\frac{c\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2),x)
```

```
[Out] (x*(a + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(c*x^2)/a))/((c*x^2)/a +
1)^(3/2)
```

3.58 $\int \sqrt{a+cx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

[Out] 1/2*a*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(1/2)+1/2*x*(c*x^2+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {201, 223, 212}

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2],x]

[Out] (x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]
)

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+cx^2}\,dx = \frac{1}{2}x\sqrt{a+cx^2} + \frac{1}{2}a\int \frac{1}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{a+cx^2} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)$$
$$= \frac{1}{2}x\sqrt{a+cx^2} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 1.04

$$\frac{1}{2}x\sqrt{a+cx^2} - \frac{a\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2],x]
```

```
[Out] (x*Sqrt[a + c*x^2])/2 - (a*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*Sqrt[c])
```

Maple [A]
time = 0.38, size = 36, normalized size = 0.78

method	result	size
default	$\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}$	36
risch	$\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.61

$$\frac{1}{2}\,\sqrt{cx^2+a}\,x + \frac{a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x + 1/2*a*arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

Fricas [A]
time = 1.40, size = 94, normalized size = 2.04

$$\left[\frac{2\sqrt{cx^2+a}\,cx + a\sqrt{c}\,\log\left(-2\,cx^2 - 2\sqrt{cx^2+a}\,\sqrt{c}\,x - a\right)}{4\,c}, \frac{\sqrt{cx^2+a}\,cx - a\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*x^2 + a)*c*x + a*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a))/c, 1/2*(sqrt(c*x^2 + a)*c*x - a*sqrt(-c)*arctan(sqrt(-c)*x/sq
rt(c*x^2 + a)))/c]
```

Sympy [A]
time = 0.92, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{cx^2}{a}}}{2} + \frac{a\,\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + c*x**2/a)/2 + a*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c))
```

Giac [A]
time = 1.93, size = 37, normalized size = 0.80

$$\frac{1}{2}\sqrt{cx^2+a}\,x - \frac{a\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x - 1/2*a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c
)
```

Mupad [B]
time = 0.13, size = 35, normalized size = 0.76

$$\frac{x\sqrt{c\,x^2+a}}{2} + \frac{a\,\ln\left(\sqrt{c}\,x + \sqrt{c\,x^2+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2),x)
```

```
[Out] (x*(a + c*x^2)^(1/2))/2 + (a*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/(2*c^(1/2)
)
```

3.59 $\int \frac{1}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+cx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Maple [A]
time = 0.37, size = 21, normalized size = 0.84

method	result	size
default	$\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(c*x/sqrt(a*c))/sqrt(c)
```

Fricas [A]
time = 1.68, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)/sqrt(c), -sqrt(-c)*arc
tan(sqrt(-c)*x/sqrt(c*x^2 + a))/c]
```

Sympy [A]
time = 0.44, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(c)*x/sqrt(a))/sqrt(c)
```

Giac [A]
time = 1.32, size = 37, normalized size = 1.48

$$\frac{1}{2}\sqrt{cx^2+a}\,x - \frac{a\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x - 1/2*a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c
)
```

Mupad [B]
time = 0.19, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(1/2),x)
```

```
[Out] log(c^(1/2)*x + (a + c*x^2)^(1/2))/c^(1/2)
```

3.60 $\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+cx^2}}$$

```
[Out] x/a/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {197}

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{x}{a\sqrt{a+cx^2}}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + c*x^2])
```

Maple [A]
time = 0.37, size = 15, normalized size = 0.94

method	result	size
gosper	$\frac{x}{a\sqrt{c\,x^2+a}}$	15
default	$\frac{x}{a\sqrt{c\,x^2+a}}$	15
trager	$\frac{x}{a\sqrt{c\,x^2+a}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/a/(c*x^2+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{cx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(c*x^2 + a)*a)
```

Fricas [A]
time = 2.10, size = 23, normalized size = 1.44

$$\frac{\sqrt{cx^2+a}\,x}{acx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2 + a)*x/(a*c*x^2 + a^2)
```

Sympy [A]
time = 0.35, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 1.33, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{cx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(c*x^2 + a)*a)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.88

$$\frac{x}{a\,\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(3/2),x)
```

```
[Out] x/(a*(a + c*x^2)^(1/2))
```

3.61 $\int \frac{1}{(a+cx^2)^{5/2}} \, dx$

Optimal. Leaf size=39

$$\frac{x}{3a\left(a+cx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+cx^2}}$$

```
[Out] 1/3*x/a/(c*x^2+a)^(3/2)+2/3*x/a^2/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {198, 197}

$$\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx &= \frac{x}{3a\left(a+cx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{3a}\\ &= \frac{x}{3a\left(a+cx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.74

$$\frac{3ax+2cx^3}{3a^2\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-5/2),x]
```

```
[Out] (3*a*x + 2*c*x^3)/(3*a^2*(a + c*x^2)^(3/2))
```

Maple [A]
time = 0.37, size = 32, normalized size = 0.82

method	result	size
gosper	$\frac{x(2cx^2+3a)}{3(cx^2+a)^{\frac{3}{2}}a^2}$	26
trager	$\frac{x(2cx^2+3a)}{3(cx^2+a)^{\frac{3}{2}}a^2}$	26
default	$\frac{x}{3a(cx^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{cx^2+a}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x/a/(c*x^2+a)^(3/2)+2/3*x/a^2/(c*x^2+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{cx^2+a}\,a^2}+\frac{x}{3\,(cx^2+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(c*x^2 + a)*a^2) + 1/3*x/((c*x^2 + a)^(3/2)*a)
```

Fricas [A]
time = 1.65, size = 47, normalized size = 1.21

$$\frac{(2\,cx^3+3\,ax)\sqrt{cx^2+a}}{3\,(a^2c^2x^4+2\,a^3cx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*c*x^3 + 3*a*x)*sqrt(c*x^2 + a)/(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.

time = 0.43, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}+\frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2/a)
) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*
x**2/a))
```

Giac [A]
time = 1.99, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,cx^2}{a^2}+\frac{3}{a}\right)}{3\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*c*x^2/a^2 + 3/a)/(c*x^2 + a)^(3/2)
```

Mupad [B]
time = 0.19, size = 28, normalized size = 0.72

$$\frac{2\,x\,\left(c\,x^2+a\right)+a\,x}{3\,a^2\,{\left(c\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(5/2),x)
```

```
[Out] (2*x*(a + c*x^2) + a*x)/(3*a^2*(a + c*x^2)^(3/2))
```

3.62 $\int \frac{1}{(a+cx^2)^{7/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x}{5a\left(a+cx^2\right)^{5/2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{8x}{15a^3\sqrt{a+cx^2}}$$

```
[Out] 1/5*x/a/(c*x^2+a)^(5/2)+4/15*x/a^2/(c*x^2+a)^(3/2)+8/15*x/a^3/(c*x^2+a)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {198, 197}

$$\frac{8x}{15a^3\sqrt{a+cx^2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{x}{5a\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-7/2),x]
```

```
[Out] x/(5*a*(a + c*x^2)^(5/2)) + (4*x)/(15*a^2*(a + c*x^2)^(3/2)) + (8*x)/(15*a^
3*Sqrt[a + c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx &= \frac{x}{5a\left(a+cx^2\right)^{5/2}}+\frac{4\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{5a}\\ &= \frac{x}{5a\left(a+cx^2\right)^{5/2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{15a^2}\\ &= \frac{x}{5a\left(a+cx^2\right)^{5/2}}+\frac{4x}{15a^2\left(a+cx^2\right)^{3/2}}+\frac{8x}{15a^3\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.69

$$\frac{15a^2x + 20acx^3 + 8c^2x^5}{15a^3 (a + cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-7/2),x]
[Out] (15*a^2*x + 20*a*c*x^3 + 8*c^2*x^5)/(15*a^3*(a + c*x^2)^(5/2))
```

Maple [A]
time = 0.43, size = 53, normalized size = 0.91

method	result	size
gosper	$\frac{x(8c^2x^4+20c\,x^2a+15a^2)}{15(c\,x^2+a)^{\frac{5}{2}}a^3}$	37
trager	$\frac{x(8c^2x^4+20c\,x^2a+15a^2)}{15(c\,x^2+a)^{\frac{5}{2}}a^3}$	37
default	$\frac{x}{5a(c\,x^2+a)^{\frac{5}{2}}} + \frac{\frac{4x}{15a(c\,x^2+a)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{c\,x^2+a}}}{a}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/5*x/a/(c*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(c*x^2+a)^(3/2)+2/3*x/a^2/(c*x^2+a)^
(1/2))
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.79

$$\frac{8\,x}{15\,\sqrt{cx^2 + a}\,a^3} + \frac{4\,x}{15\,(cx^2 + a)^{\frac{3}{2}}a^2} + \frac{x}{5\,(cx^2 + a)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="maxima")
[Out] 8/15*x/(sqrt(c*x^2 + a)*a^3) + 4/15*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*x/((c*x
^2 + a)^(5/2)*a)
```

Fricas [A]
time = 2.26, size = 69, normalized size = 1.19

$$\frac{(8\,c^2x^5 + 20\,acx^3 + 15\,a^2x)\sqrt{cx^2 + a}}{15\,(a^3c^3x^6 + 3\,a^4c^2x^4 + 3\,a^5cx^2 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*c^2*x^5 + 20*a*c*x^3 + 15*a^2*x)*sqrt(c*x^2 + a)/(a^3*c^3*x^6 + 3*a
^4*c^2*x^4 + 3*a^5*c*x^2 + a^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(51) = 102$.
time = 0.73, size = 413, normalized size = 7.12

$$\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{35a^4cx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{28a^3c^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{8a^2c^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(7/2),x)
```

```
[Out] 15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c
*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x*
*6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a) +
45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*
x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5/(1
5*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 4
5*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 +
c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/
2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) +
15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 1.02, size = 41, normalized size = 0.71

$$\frac{\left(4\,x^2\left(\frac{2\,c^2x^2}{a^3}+\frac{5\,c}{a^2}\right)+\frac{15}{a}\right)x}{15\,(cx^2+a)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*x^2*(2*c^2*x^2/a^3 + 5*c/a^2) + 15/a)*x/(c*x^2 + a)^(5/2)
```

Mupad [B]
time = 0.19, size = 44, normalized size = 0.76

$$\frac{8\,x\,{\left(c\,x^2+a\right)}^2+3\,a^2\,x+4\,a\,x\,\left(c\,x^2+a\right)}{15\,a^3\,{\left(c\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(7/2),x)
```

```
[Out] (8*x*(a + c*x^2)^2 + 3*a^2*x + 4*a*x*(a + c*x^2))/(15*a^3*(a + c*x^2)^(5/2)
)
```

3.63 $\int \frac{1}{\left(a+cx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=77

$$\frac{x}{7a\left(a+cx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{16x}{35a^4\sqrt{a+cx^2}}$$

```
[Out] 1/7*x/a/(c*x^2+a)^(7/2)+6/35*x/a^2/(c*x^2+a)^(5/2)+8/35*x/a^3/(c*x^2+a)^(3/
2)+16/35*x/a^4/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {198, 197}

$$\frac{16x}{35a^4\sqrt{a+cx^2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{x}{7a\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(-9/2),x]
```

```
[Out] x/(7*a*(a + c*x^2)^(7/2)) + (6*x)/(35*a^2*(a + c*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + c*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+cx^2)^{9/2}}\,dx = \frac{x}{7a\,(a+cx^2)^{7/2}} + \frac{6\int \frac{1}{(a+cx^2)^{7/2}}\,dx}{7a}$$
$$= \frac{x}{7a\,(a+cx^2)^{7/2}} + \frac{6x}{35a^2\,(a+cx^2)^{5/2}} + \frac{24\int \frac{1}{(a+cx^2)^{5/2}}\,dx}{35a^2}$$
$$= \frac{x}{7a\,(a+cx^2)^{7/2}} + \frac{6x}{35a^2\,(a+cx^2)^{5/2}} + \frac{8x}{35a^3\,(a+cx^2)^{3/2}} + \frac{16\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{35a^3}$$
$$= \frac{x}{7a\,(a+cx^2)^{7/2}} + \frac{6x}{35a^2\,(a+cx^2)^{5/2}} + \frac{8x}{35a^3\,(a+cx^2)^{3/2}} + \frac{16x}{35a^4\sqrt{a+cx^2}}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.66

$$\frac{35a^3x + 70a^2cx^3 + 56ac^2x^5 + 16c^3x^7}{35a^4\,(a+cx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(-9/2),x]
```

```
[Out] (35*a^3*x + 70*a^2*c*x^3 + 56*a*c^2*x^5 + 16*c^3*x^7)/(35*a^4*(a + c*x^2)^(
7/2))
```

Maple [A]
time = 0.41, size = 74, normalized size = 0.96

method	result	size
gosper	$\frac{x\left(16c^3x^6+56a\,c^2x^4+70a^2c\,x^2+35a^3\right)}{35\left(c\,x^2+a\right)^{\frac{7}{2}}a^4}$	48
trager	$\frac{x\left(16c^3x^6+56a\,c^2x^4+70a^2c\,x^2+35a^3\right)}{35\left(c\,x^2+a\right)^{\frac{7}{2}}a^4}$	48
default	$\frac{x}{7a\left(c\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(c\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(c\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{c\,x^2+a}}\right)}{7a}}{a}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x/a/(c*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(c*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(c*x^
2+a)^(3/2)+2/3*x/a^2/(c*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 61, normalized size = 0.79

$$\frac{16\,x}{35\,\sqrt{cx^2+a}\,a^4}+\frac{8\,x}{35\,(cx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,x}{35\,(cx^2+a)^{\frac{5}{2}}a^2}+\frac{x}{7\,(cx^2+a)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*x/(sqrt(c*x^2 + a)*a^4) + 8/35*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*x/((c
*x^2 + a)^(5/2)*a^2) + 1/7*x/((c*x^2 + a)^(7/2)*a)
```

Fricas [A]
time = 1.56, size = 91, normalized size = 1.18

$$\frac{(16\,c^3x^7+56\,ac^2x^5+70\,a^2cx^3+35\,a^3x)\sqrt{cx^2+a}}{35\,(a^4c^4x^8+4\,a^5c^3x^6+6\,a^6c^2x^4+4\,a^7cx^2+a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^7 + 56*a*c^2*x^5 + 70*a^2*c*x^3 + 35*a^3*x)*sqrt(c*x^2 + a)/
(a^4*c^4*x^8 + 4*a^5*c^3*x^6 + 6*a^6*c^2*x^4 + 4*a^7*c*x^2 + a^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. 2(70) = 140.
time = 1.04, size = 1265, normalized size = 16.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(9/2),x)
```

```
[Out] 35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 +
 c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**
3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 21
0*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1
+ c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(
35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a
) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sq
rt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/
2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37/2)*sqrt(
1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**
2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 52
5*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1
```

```
+ c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c**3*
x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**
2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6
*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(
27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x*
*2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/
2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) +
 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(
1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*
c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**11/(35*a**(37/2)*sqrt(1 +
 c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x
**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a
**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c
*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x**13
/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a)
 + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqr
t(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2
)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a
))
```

Giac [A]
time = 0.84, size = 55, normalized size = 0.71

$$\frac{\left(2\left(4\,x^2\left(\frac{2\,c^3x^2}{a^4}+\frac{7\,c^2}{a^3}\right)+\frac{35\,c}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\,(cx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*c^3*x^2/a^4 + 7*c^2/a^3) + 35*c/a^2)*x^2 + 35/a)*x/(c*x^2
 + a)^(7/2)
```

Mupad [B]
time = 0.20, size = 61, normalized size = 0.79

$$\frac{16\,x}{35\,a^4\,\sqrt{c\,x^2+a}}+\frac{8\,x}{35\,a^3\,{\left(c\,x^2+a\right)}^{3/2}}+\frac{6\,x}{35\,a^2\,{\left(c\,x^2+a\right)}^{5/2}}+\frac{x}{7\,a\,{\left(c\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(9/2),x)
```

```
[Out] (16*x)/(35*a^4*(a + c*x^2)^(1/2)) + (8*x)/(35*a^3*(a + c*x^2)^(3/2)) + (6*x
)/(35*a^2*(a + c*x^2)^(5/2)) + x/(7*a*(a + c*x^2)^(7/2))
```

3.64 $\int \left(4+12x+9x^2\right)^{3/2}\, dx$

Optimal. Leaf size=23

$$\frac{1}{12}(2+3x)\left(4+12x+9x^2\right)^{3/2}$$

```
[Out] 1/12*(2+3*x)*(9*x^2+12*x+4)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {623}

$$\frac{1}{12}(3x+2)\left(9x^2+12x+4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 12*x + 9*x^2)^(3/2),x]
[Out] ((2 + 3*x)*(4 + 12*x + 9*x^2)^(3/2))/12
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \left(4+12x+9x^2\right)^{3/2}\, dx=\frac{1}{12}(2+3x)\left(4+12x+9x^2\right)^{3/2}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{12}(2+3x)\left((2+3x)^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 12*x + 9*x^2)^(3/2),x]
[Out] ((2 + 3*x)*((2 + 3*x)^2)^(3/2))/12
```

Maple [A]
time = 0.43, size = 17, normalized size = 0.74

method	result	size
default	$\frac{(2+3x)\left((2+3x)^2\right)^{\frac{3}{2}}}{12}$	17
risch	$\frac{\sqrt{(2+3x)^2}\,(2+3x)^3}{12}$	19
gosper	$\frac{x(27x^3+72x^2+72x+32)\left((2+3x)^2\right)^{\frac{3}{2}}}{4(2+3x)^3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+12*x+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(2+3*x)*((2+3*x)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 30, normalized size = 1.30

$$\frac{1}{4}\left(9\,x^2+12\,x+4\right)^{\frac{3}{2}}x+\frac{1}{6}\left(9\,x^2+12\,x+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 12*x + 4)^(3/2)*x + 1/6*(9*x^2 + 12*x + 4)^(3/2)
```

Fricas [A]
time = 2.15, size = 19, normalized size = 0.83

$$\frac{27}{4}\,x^4+18\,x^3+18\,x^2+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 27/4*x^4 + 18*x^3 + 18*x^2 + 8*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(9x^2+12x+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+12*x+4)**(3/2),x)
```

```
[Out] Integral((9*x**2 + 12*x + 4)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.
time = 2.44, size = 45, normalized size = 1.96

$$\frac{3}{4}\left(3\,x^2+4\,x\right)^2\mathrm{sgn}(3\,x+2)+2\left(3\,x^2+4\,x\right)\mathrm{sgn}(3\,x+2)+\frac{4}{3}\,\mathrm{sgn}(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*(3*x^2 + 4*x)^2*sgn(3*x + 2) + 2*(3*x^2 + 4*x)*sgn(3*x + 2) + 4/3*sgn(3
*x + 2)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 0.83

$$\frac{(9\,x+6)\,\left(9\,x^2+12\,x+4\right)^{3/2}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 9*x^2 + 4)^(3/2),x)
```

```
[Out] ((9*x + 6)*(12*x + 9*x^2 + 4)^(3/2))/36
```

3.65 $\int \sqrt{4+12x+9x^2}\, dx$

Optimal. Leaf size=23

$$\frac{1}{6}(2+3x)\sqrt{4+12x+9x^2}$$

```
[Out] 1/6*(2+3*x)*((2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{623\}$

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{4+12x+9x^2}\, dx = \frac{1}{6}(2+3x)\sqrt{4+12x+9x^2}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.09

$$\frac{x\sqrt{(2+3x)^2}\,(4+3x)}{4+6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] (x*Sqrt[(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.40, size = 16, normalized size = 0.70

method	result	size
default	$\frac{\operatorname{csgn}(2+3x)(2+3x)^2}{6}$	16
gosper	$\frac{x(3x+4)\sqrt{(2+3x)^2}}{4+6x}$	25
risch	$\frac{3\sqrt{(2+3x)^2}\,x^2}{2(2+3x)}+\frac{2\sqrt{(2+3x)^2}\,x}{2+3x}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+12*x+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*csgn(2+3*x)*(2+3*x)^2
```

Maxima [A]
time = 0.51, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{9x^2+12x+4}\,x+\frac{1}{3}\sqrt{9x^2+12x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(9*x^2 + 12*x + 4)*x + 1/3*sqrt(9*x^2 + 12*x + 4)
```

Fricas [A]
time = 0.84, size = 9, normalized size = 0.39

$$\frac{3}{2}x^2+2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="fricas")
[Out] 3/2*x^2 + 2*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{9x^2+12x+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+12*x+4)**(1/2),x)
[Out] Integral(sqrt(9*x**2 + 12*x + 4), x)
```

Giac [A]
time = 1.64, size = 26, normalized size = 1.13

$$\frac{1}{2}\left(3\,x^{2}+4\,x\right)\mathrm{sgn}(3\,x+2)+\frac{2}{3}\,\mathrm{sgn}(3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="giac")
[Out] 1/2*(3*x^2 + 4*x)*sgn(3*x + 2) + 2/3*sgn(3*x + 2)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 0.83

$$\frac{(3\,x+2)\,\sqrt{9\,x^{2}+12\,x+4}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 9*x^2 + 4)^(1/2),x)
[Out] ((3*x + 2)*(12*x + 9*x^2 + 4)^(1/2))/6
```

3.66 $\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{(2+3x)\log(2+3x)}{3\sqrt{4+12x+9x^2}}$$

```
[Out] 1/3*(2+3*x)*ln(2+3*x)/((2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {622, 31}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4+12x+9x^2}}\,dx &= \frac{(6+9x)\int \frac{1}{6+9x}\,dx}{\sqrt{4+12x+9x^2}}\\ &= \frac{(2+3x)\log(2+3x)}{3\sqrt{4+12x+9x^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.90

$$\frac{(2+3x)\log(2+3x)}{3\sqrt{(2+3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[(2 + 3*x)^2])
```

Maple [A]
time = 0.45, size = 23, normalized size = 0.79

method	result	size
meijerg	$\frac{\ln\left(1+\frac{3x}{2}\right)}{3}$	9
default	$\frac{(2+3x)\ln(2+3x)}{3\sqrt{(2+3x)^{2}}}$	23
risch	$\frac{\sqrt{(2+3x)^{2}}\,\ln(2+3x)}{6+9x}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+12*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2+3*x)*ln(2+3*x)/((2+3*x)^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 6, normalized size = 0.21

$$\frac{1}{3}\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x + 2/3)
```

Fricas [A]
time = 1.50, size = 8, normalized size = 0.28

$$\frac{1}{3}\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{9x^{2}+12x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+12*x+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 12*x + 4), x)
```

Giac [A]
time = 1.42, size = 25, normalized size = 0.86

$$\frac{\log\left(|3\,x+2||\mathrm{sgn}(3\,x+2)|\right)}{3\,\mathrm{sgn}\left(3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2)*abs(sgn(3*x + 2)))/sgn(3*x + 2)
```

Mupad [B]
time = 0.28, size = 14, normalized size = 0.48

$$\frac{\ln\left(9\,x+6\right)\,\mathrm{sign}(18\,x+12)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(12*x + 9*x^2 + 4)^(1/2),x)
```

```
[Out] (log(9*x + 6)*sign(18*x + 12))/3
```

3.67 $\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{6(2+3x)\sqrt{4+12x+9x^2}}$$

```
[Out] -1/6/(2+3*x)/((2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{621\}$

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 12*x + 9*x^2)^(-3/2),x]
[Out] -1/6*1/((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx = -\frac{1}{6(2+3x)\sqrt{4+12x+9x^2}}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.80

$$-\frac{2+3x}{6\left((2+3x)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 12*x + 9*x^2)^(-3/2),x]
[Out] -1/6*(2 + 3*x)/((2 + 3*x)^2)^(3/2)
```

Maple [A]
time = 0.43, size = 17, normalized size = 0.68

method	result	size
meijerg	$\frac{x\left(2+\frac{3x}{2}\right)}{16\left(1+\frac{3x}{2}\right)^{2}}$	16
gosper	$-\frac{2+3x}{6\left(\left(2+3x\right)^{2}\right)^{\frac{3}{2}}}$	17
default	$-\frac{2+3x}{6\left(\left(2+3x\right)^{2}\right)^{\frac{3}{2}}}$	17
risch	$-\frac{\sqrt{\left(2+3x\right)^{2}}}{6\left(2+3x\right)^{3}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+12*x+4)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(2+3*x)/((2+3*x)^2)^(3/2)
```

Maxima [A]
time = 0.51, size = 9, normalized size = 0.36

$$-\frac{1}{6\,{\left(3\,x+2\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="maxima")
[Out] -1/6/(3*x + 2)^2
```

Fricas [A]
time = 2.01, size = 14, normalized size = 0.56

$$-\frac{1}{6\,{\left(9\,x^{2}+12\,x+4\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="fricas")
[Out] -1/6/(9*x^2 + 12*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(9x^{2}+12x+4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+12*x+4)**(3/2),x)
```

```
[Out] Integral((9*x**2 + 12*x + 4)**(-3/2), x)
```

Giac [A]
time = 1.49, size = 17, normalized size = 0.68

$$-\frac{1}{6\left(3\,x+2\right)^{2}\mathrm{sgn}\left(3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6/((3*x + 2)^2*sgn(3*x + 2))
```

Mupad [B]
time = 0.17, size = 21, normalized size = 0.84

$$-\frac{\sqrt{9\,x^2+12\,x+4}}{6\,\left(3\,x+2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(12*x + 9*x^2 + 4)^(3/2),x)
```

```
[Out] -(12*x + 9*x^2 + 4)^(1/2)/(6*(3*x + 2)^3)
```

3.68 $\int \sqrt{4-12x+9x^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{4-12x+9x^2}$$

```
[Out] -1/6*(2-3*x)*((-2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {623}

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] -1/6*((2 - 3*x)*Sqrt[4 - 12*x + 9*x^2])
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{4-12x+9x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{4-12x+9x^2}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.09

$$\frac{\sqrt{(2-3x)^2}\,x(-4+3x)}{-4+6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] (Sqrt[(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.07, size = 16, normalized size = 0.70

method	result	size
default	$\frac{\mathrm{csgn}(-2+3x)(-2+3x)^2}{6}$	16
gosper	$\frac{x(3x-4)\sqrt{(-2+3x)^2}}{-4+6x}$	25
risch	$\frac{3\sqrt{(-2+3x)^2}\,x^2}{2(-2+3x)}-\frac{2\sqrt{(-2+3x)^2}\,x}{-2+3x}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*csgn(-2+3*x)*(-2+3*x)^2
```

Maxima [A]
time = 0.51, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{9x^2-12x+4}\,x-\frac{1}{3}\sqrt{9x^2-12x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 - 12*x + 4)*x - 1/3*sqrt(9*x^2 - 12*x + 4)
```

Fricas [A]
time = 1.38, size = 9, normalized size = 0.39

$$\frac{3}{2}x^2-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 - 2*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.35

$$\frac{3x^2}{2}-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)**2)**(1/2),x)
```

```
[Out] 3*x**2/2 - 2*x
```

Giac [A]
time = 1.34, size = 26, normalized size = 1.13

$$\frac{1}{2}\left(3\,x^2-4\,x\right)\mathrm{sgn}(3\,x-2)+\frac{2}{3}\,\mathrm{sgn}(3\,x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2+3*x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(3*x^2 - 4*x)*sgn(3*x - 2) + 2/3*sgn(3*x - 2)
```

Mupad [B]
time = 0.11, size = 13, normalized size = 0.57

$$\frac{|3\,x-2|\,\left(3\,x-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x - 2)^2)^(1/2),x)
[Out] (abs(3*x - 2)*(3*x - 2))/6
```

3.69 $\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{4-12x+9x^2}}$$

```
[Out] -1/3*(2-3*x)*ln(2-3*x)/((-2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {622, 31}

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] -1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[4 - 12*x + 9*x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx &= \frac{(-6+9x)\int \frac{1}{-6+9x}\,dx}{\sqrt{4-12x+9x^2}}\\ &= -\frac{(2-3x)\log(2-3x)}{3\sqrt{4-12x+9x^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.90

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] -1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[(2 - 3*x)^2]
```

Maple [A]
time = 0.37, size = 23, normalized size = 0.79

method	result	size
default	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{(-2+3x)^2}}$	23
risch	$\frac{\sqrt{(-2+3x)^2}\,\ln(-2+3x)}{-6+9x}$	25
meijerg	$-\frac{2\ln\left(1-\frac{3x}{2}\right)}{3\sqrt{(-2+3x)^2}}+\frac{x\ln\left(1-\frac{3x}{2}\right)}{\sqrt{(-2+3x)^2}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-2+3*x)/((-2+3*x)^2)^(1/2)*ln(-2+3*x)
```

Maxima [A]
time = 0.48, size = 6, normalized size = 0.21

$$\frac{1}{3}\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x - 2/3)
```

Fricas [A]
time = 1.66, size = 8, normalized size = 0.28

$$\frac{1}{3}\log\left(3x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x - 2)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.24

$$\frac{\log (3x-2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)**2)**(1/2),x)
```

```
[Out] log(3*x - 2)/3
```

Giac [A]
time = 1.50, size = 15, normalized size = 0.52

$$\frac{1}{3}\log\left(|3x-2|\right)\operatorname{sgn}(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x - 2))*sgn(3*x - 2)
```

Mupad [B]
time = 0.35, size = 14, normalized size = 0.48

$$\frac{\ln(3x-2)\ \operatorname{sign}(3x-2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x - 2)^2)^(1/2),x)
```

```
[Out] (log(3*x - 2)*sign(3*x - 2))/3
```

3.70 $\int \sqrt{-4+12x-9x^2}\, dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{-4+12x-9x^2}$$

```
[Out] -1/6*(2-3*x)*(-(-2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {623}

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] -1/6*((2 - 3*x)*Sqrt[-4 + 12*x - 9*x^2])
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{-4+12x-9x^2}\, dx = -\frac{1}{6}(2-3x)\sqrt{-4+12x-9x^2}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.17

$$\frac{\sqrt{-(2-3x)^2}\,x(-4+3x)}{-4+6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] (Sqrt[-(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

Maple [A]
time = 0.52, size = 27, normalized size = 1.17

method	result	size
gosper	$\frac{x(3x-4)\sqrt{-(-2+3x)^{2}}}{-4+6x}$	27
default	$\frac{x(3x-4)\sqrt{-(-2+3x)^{2}}}{-4+6x}$	27
risch	$-\frac{2\sqrt{-(-2+3x)^{2}}\,x}{-2+3x}+\frac{3\sqrt{-(-2+3x)^{2}}\,x^{2}}{2(-2+3x)}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(3*x-4)*(-(-2+3*x)^2)^(1/2)/(-2+3*x)
```

Maxima [A]
time = 0.48, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{-9x^{2}+12x-4}\,x-\frac{1}{3}\sqrt{-9x^{2}+12x-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 + 12*x - 4)*x - 1/3*sqrt(-9*x^2 + 12*x - 4)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.45, size = 9, normalized size = 0.39

$$\frac{3}{2}i\,x^{2}-2i\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*I*x^2 - 2*I*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-(3x-2)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(3*x - 2)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.98, size = 26, normalized size = 1.13

$$-\frac{1}{2}i\left(3\,x^{2}-4\,x\right)\mathrm{sgn}(-3\,x+2)-\frac{2}{3}i\,\mathrm{sgn}(-3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*I*(3*x^2 - 4*x)*sgn(-3*x + 2) - 2/3*I*sgn(-3*x + 2)
```

Mupad [B]
time = 0.33, size = 18, normalized size = 0.78

$$\frac{(3\,x-2)\,\sqrt{-(3\,x-2)^{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(3*x - 2)^2)^(1/2),x)
[Out] ((3*x - 2)*(-(3*x - 2)^2)^(1/2))/6
```

3.71 $\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-4+12x-9x^2}}$$

```
[Out] -1/3*(2-3*x)*ln(2-3*x)/(-(-2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {622, 31}

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] -1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[-4 + 12*x - 9*x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx &= \frac{(6-9x)\int \frac{1}{6-9x}\,dx}{\sqrt{-4+12x-9x^2}}\\ &= -\frac{(2-3x)\log(2-3x)}{3\sqrt{-4+12x-9x^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 0.97

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-4 + 12*x - 9*x^2],x]
[Out] -1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[-(2 - 3*x)^2]
```

Maple [A]
time = 0.44, size = 25, normalized size = 0.86

method	result	size
meijerg	$-\frac{i \ln\left(1-\frac{3x}{2}\right)}{3}$	10
default	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{-\left(-2+3x\right)^{2}}}$	25
risch	$\frac{(-2+3x)\ln(-2+3x)}{3\sqrt{-\left(-2+3x\right)^{2}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(-2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/(-(-2+3*x)^2)^(1/2)*(-2+3*x)*ln(-2+3*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 6, normalized size = 0.21

$$\frac{1}{3} i \log\left(x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*I*log(x - 2/3)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.46, size = 6, normalized size = 0.21

$$-\frac{1}{3} i \log\left(x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*I*log(x - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(3x-2\right)^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-(3*x - 2)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.77, size = 23, normalized size = 0.79

$$\frac{i \log \left((-3 i x+2 i) \operatorname{sgn}(-3 x+2)\right)}{3 \operatorname{sgn}(-3 x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*I*log((-3*I*x + 2*I)*sgn(-3*x + 2))/sgn(-3*x + 2)
```

Mupad [B]
time = 0.25, size = 15, normalized size = 0.52

$$-\frac{\ln (2-3 x) \operatorname{sign}(3 x-2) \; 1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x - 2)^2)^(1/2),x)
```

```
[Out] -(log(2 - 3*x)*sign(3*x - 2)*1i)/3
```

3.72 $\int \sqrt{-4-12x-9x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{6}(2+3x)\sqrt{-4-12x-9x^2}$$

```
[Out] 1/6*(2+3*x)*(-(2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {623}

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-4 - 12*x - 9*x^2],x]
[Out] ((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \sqrt{-4-12x-9x^2}\,dx=\frac{1}{6}(2+3x)\sqrt{-4-12x-9x^2}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.17

$$\frac{x\sqrt{-(2+3x)^2}\,(4+3x)}{4+6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-4 - 12*x - 9*x^2],x]
[Out] (x*Sqrt[-(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

Maple [A]
time = 0.40, size = 27, normalized size = 1.17

method	result	size
gosper	$\frac{x(3x+4)\sqrt{-(2+3x)^2}}{4+6x}$	27
default	$\frac{x(3x+4)\sqrt{-(2+3x)^2}}{4+6x}$	27
risch	$\frac{2\sqrt{-(2+3x)^2}\,x}{2+3x}+\frac{3\sqrt{-(2+3x)^2}\,x^2}{2(2+3x)}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(3*x+4)*(-(2+3*x)^2)^(1/2)/(2+3*x)
```

Maxima [A]
time = 0.48, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{-9\,x^2-12\,x-4}\,x+\frac{1}{3}\sqrt{-9\,x^2-12\,x-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 - 12*x - 4)*x + 1/3*sqrt(-9*x^2 - 12*x - 4)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.32, size = 9, normalized size = 0.39

$$\frac{3}{2}i\,x^2+2i\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*I*x^2 + 2*I*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-(3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(3*x + 2)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.83, size = 26, normalized size = 1.13

$$-\frac{1}{2}i\left(3x^2+4x\right)\operatorname{sgn}(-3x-2)-\frac{2}{3}i\operatorname{sgn}(-3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*(3*x^2 + 4*x)*sgn(-3*x - 2) - 2/3*I*sgn(-3*x - 2)
```

Mupad [B]
time = 0.07, size = 18, normalized size = 0.78

$$\frac{(3x+2)\sqrt{-(3x+2)^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(3*x + 2)^2)^(1/2),x)
```

```
[Out] ((3*x + 2)*(-(3*x + 2)^2)^(1/2))/6
```

3.73 $\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{(2+3x)\log(2+3x)}{3\sqrt{-4-12x-9x^2}}$$

```
[Out] 1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {622, 31}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx &= -\left(-\frac{(-6-9x)\int \frac{1}{-6-9x}\,dx}{\sqrt{-4-12x-9x^2}}\right)\\ &= \frac{(2+3x)\log(2+3x)}{3\sqrt{-4-12x-9x^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 0.97

$$\frac{(2+3x)\log(2+3x)}{3\sqrt{-(2+3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-4 - 12*x - 9*x^2],x]
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Maple [A]
time = 0.43, size = 25, normalized size = 0.86

method	result	size
meijerg	$-\frac{i\ln\left(1+\frac{3x}{2}\right)}{3}$	10
default	$\frac{(2+3x)\ln(2+3x)}{3\sqrt{-(2+3x)^{2}}}$	25
risch	$\frac{(2+3x)\ln(2+3x)}{3\sqrt{-(2+3x)^{2}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 6, normalized size = 0.21

$$\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*I*log(x + 2/3)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.39, size = 6, normalized size = 0.21

$$-\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*I*log(x + 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-(3x+2)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-(3*x + 2)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.67, size = 23, normalized size = 0.79

$$\frac{i\,\log\left((-3i\,x-2i)\mathrm{sgn}(-3\,x-2)\right)}{3\,\mathrm{sgn}\left(-3\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*I*log((-3*I*x - 2*I)*sgn(-3*x - 2))/sgn(-3*x - 2)
```

Mupad [B]
time = 0.29, size = 15, normalized size = 0.52

$$-\frac{\ln\left(-3\,x-2\right)\,\mathrm{sign}(3\,x+2)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x + 2)^2)^(1/2),x)
```

```
[Out] -(log(- 3*x - 2)*sign(3*x + 2)*1i)/3
```

3.74 $\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal. Leaf size=109

$$\frac{(1-b-2cx)^6}{384c^6}-\frac{5(1-b-2cx)^7}{896c^6}+\frac{5(1-b-2cx)^8}{1024c^6}-\frac{5(1-b-2cx)^9}{2304c^6}+\frac{(1-b-2cx)^{10}}{2048c^6}-\frac{(1-b-2cx)^{11}}{22528c^6}$$

```
[Out] 1/384*(-2*c*x-b+1)^6/c^6-5/896*(-2*c*x-b+1)^7/c^6+5/1024*(-2*c*x-b+1)^8/c^6
-5/2304*(-2*c*x-b+1)^9/c^6+1/2048*(-2*c*x-b+1)^10/c^6-1/22528*(-2*c*x-b+1)^
11/c^6
```

Rubi [A]

time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {624, 45}

$$-\frac{(-b-2cx+1)^{11}}{22528c^6}+\frac{(-b-2cx+1)^{10}}{2048c^6}-\frac{5(-b-2cx+1)^9}{2304c^6}+\frac{5(-b-2cx+1)^8}{1024c^6}-\frac{5(-b-2cx+1)^7}{896c^6}+\frac{(-b-2cx+1)^6}{384c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (1 - b - 2*c*x)^6/(384*c^6) - (5*(1 - b - 2*c*x)^7)/(896*c^6) + (5*(1 - b -
 2*c*x)^8)/(1024*c^6) - (5*(1 - b - 2*c*x)^9)/(2304*c^6) + (1 - b - 2*c*x)^
10/(2048*c^6) - (1 - b - 2*c*x)^11/(22528*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 624

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-1+b)+cx\right)^5 \left(\frac{1+b}{2}+cx\right)^5 dx}{c^5}$$
$$= \frac{\int \left(\left(\frac{1}{2}(-1+b)+cx\right)^5+5\left(\frac{1}{2}(-1+b)+cx\right)^6+10\left(\frac{1}{2}(-1+b)+cx\right)^7+10\left(\right.}{c^5}$$
$$= \frac{(1-b-2cx)^6}{384c^6}-\frac{5(1-b-2cx)^7}{896c^6}+\frac{5(1-b-2cx)^8}{1024c^6}-\frac{5(1-b-2cx)^9}{2304c^6}+$$

Mathematica [A]
time = 0.02, size = 207, normalized size = 1.90

$$\frac{(-1+b^2)^5 x}{1024c^5}+\frac{5b(-1+b^2)^4 x^2}{512c^4}+\frac{5(-1+b^2)^3(-1+9b^2) x^3}{768c^3}+\frac{5b(-1+b^2)^2(-1+3b^2) x^4}{64c^2}+\frac{(-1+b^2)(1-14b^2+21b^4) x^5}{32c}+\frac{1}{48}b(15-70b^2+63b^4) x^6+\frac{5}{56}(1-14b^2+21b^4) cx^7+\frac{5}{8}(-b+3b^3) c^2x^8+\frac{5}{36}(-1+9b^2) c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-1 + b^2)^5*x)/(1024*c^5) + (5*b*(-1 + b^2)^4*x^2)/(512*c^4) + (5*(-1 + b
^2)^3*(-1 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-1 + b^2)^2*(-1 + 3*b^2)*x^4)/(64
*c^2) + ((-1 + b^2)*(1 - 14*b^2 + 21*b^4)*x^5)/(32*c) + (b*(15 - 70*b^2 + 6
3*b^4)*x^6)/48 + (5*(1 - 14*b^2 + 21*b^4)*c*x^7)/56 + (5*(-b + 3*b^3)*c^2*x
^8)/8 + (5*(-1 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(97) = 194$.
time = 0.62, size = 636, normalized size = 5.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-1)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-1)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-1
/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-1)*c^2*b+b*(2*(3/2*b^2-1/2)*c^2+4*b^2*c^2
)+c*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c))*x^8+1/7*(1/4*(b^2-1)/c*(2*(3/2*b^2-1
/2)*c^2+4*b^2*c^2)+b*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c)+c*(1/8*(b^2-1)^2+2*(
b^2-1)*b^2+(3/2*b^2-1/2)^2))*x^7+1/6*(1/4*(b^2-1)/c*((b^2-1)*c*b+4*(3/2*b^2
-1/2)*b*c)+b*(1/8*(b^2-1)^2+2*(b^2-1)*b^2+(3/2*b^2-1/2)^2)+c*(1/4*(b^2-1)^2
/c*b+(b^2-1)/c*b*(3/2*b^2-1/2)))*x^6+1/5*(1/4*(b^2-1)/c*(1/8*(b^2-1)^2+2*(b
^2-1)*b^2+(3/2*b^2-1/2)^2)+b*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2))+
c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2))*x^5+1/4*(1/4*(b^
2-1)/c*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2))+b*(1/8*(b^2-1)^2/c^2*(
3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16/c^2*(b^2-1)^3*b)*x^4+1/3*(1/4*(b^2
-1)/c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16*b^2*(b^2
-1)^3/c^3+1/256/c^3*(b^2-1)^4)*x^3+5/512*(b^2-1)^4/c^4*b*x^2+1/1024*(b^2-1)
^5/c^5*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.
time = 0.30, size = 234, normalized size = 2.15

$$\frac{1}{11}\,c^5x^{11} + \frac{1}{2}\,bc^4x^{10} + \frac{10}{9}\,b^2c^3x^9 + \frac{5}{4}\,b^3c^2x^8 + \frac{5}{7}\,b^4cx^7 + \frac{1}{6}\,b^5x^6 + \frac{5\,(2\,cx^3 + 3\,bx^2)(b^2-1)^4}{1536\,c^4} + \frac{(6\,c^2x^5 + 15\,bcx^4 + 10\,b^2x^3)(b^2-1)^3}{192\,c^3} + \frac{(20\,c^3x^7 + 70\,bc^2x^6 + 84\,b^2cx^5 + 35\,b^3x^4)(b^2-1)^2}{224\,c^2} + \frac{(70\,c^4x^9 + 315\,bc^3x^8 + 540\,b^2c^2x^7 + 420\,b^3cx^6 + 126\,b^4x^5)(b^2-1)}{504\,c} + \frac{(b^2-1)^5x}{1024\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 1)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 1)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 1)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 1)/c + 1/1024*(b^2 - 1)^5*x/c^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(85) = 170$.
time = 1.47, size = 233, normalized size = 2.14

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^9x^{10} + 98560\,(9\,b^2-1)c^8x^9 + 443520\,(3\,b^3-b)c^7x^8 + 63360\,(21\,b^4-14\,b^2+1)c^6x^7 + 14784\,(63\,b^5-70\,b^3+15\,b)c^5x^6 + 22176\,(21\,b^6-35\,b^4+15\,b^2-1)c^4x^5 + 55440\,(3\,b^7-7\,b^5+5\,b^3-b)c^3x^4 + 4620\,(9\,b^8-28\,b^6+30\,b^4-12\,b^2+1)c^2x^3 + 6930\,(b^9-4\,b^7+6\,b^5-4\,b^3+b)cx^2 + 693\,(b^{10}-5\,b^8+10\,b^6-10\,b^4+5\,b^2-1)x}{709632\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 1)*c^8*x^9 +
 443520*(3*b^3 - b)*c^7*x^8 + 63360*(21*b^4 - 14*b^2 + 1)*c^6*x^7 + 14784*(
63*b^5 - 70*b^3 + 15*b)*c^5*x^6 + 22176*(21*b^6 - 35*b^4 + 15*b^2 - 1)*c^4*
x^5 + 55440*(3*b^7 - 7*b^5 + 5*b^3 - b)*c^3*x^4 + 4620*(9*b^8 - 28*b^6 + 30
*b^4 - 12*b^2 + 1)*c^2*x^3 + 6930*(b^9 - 4*b^7 + 6*b^5 - 4*b^3 + b)*c*x^2 +
 693*(b^10 - 5*b^8 + 10*b^6 - 10*b^4 + 5*b^2 - 1)*x)/c^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(95) = 190$.
time = 0.07, size = 253, normalized size = 2.32

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\cdot\left(\frac{5b^2c^3}{4} - \frac{5c^3}{36}\right) + x^8\cdot\left(\frac{15b^3c^2}{8} - \frac{5bc^2}{8}\right) + x^7\cdot\left(\frac{15b^4c}{8} - \frac{5b^2c}{4} + \frac{5c}{56}\right) + x^6\cdot\left(\frac{21b^5}{16} - \frac{35b^3}{24} + \frac{5b}{16}\right) + \frac{x^5\cdot(21b^6 - 35b^4 + 15b^2 - 1)}{32c} + \frac{x^4\cdot(15b^7 - 35b^5 + 25b^3 - 5b)}{64c^2} + \frac{x^3\cdot(45b^8 - 140b^6 + 150b^4 - 60b^2 + 5)}{768c^3} + \frac{x^2\cdot(5b^9 - 20b^7 + 30b^5 - 20b^3 + 5b)}{512c^4} + \frac{x(b^{10} - 5b^8 + 10b^6 - 10b^4 + 5b^2 - 1)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-1)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/36) + x**8*(1
5*b**3*c**2/8 - 5*b*c**2/8) + x**7*(15*b**4*c/8 - 5*b**2*c/4 + 5*c/56) + x*
*6*(21*b**5/16 - 35*b**3/24 + 5*b/16) + x**5*(21*b**6 - 35*b**4 + 15*b**2 -
 1)/(32*c) + x**4*(15*b**7 - 35*b**5 + 25*b**3 - 5*b)/(64*c**2) + x**3*(45*
```

```
b**8 - 140*b**6 + 150*b**4 - 60*b**2 + 5)/(768*c**3) + x**2*(5*b**9 - 20*b*
*7 + 30*b**5 - 20*b**3 + 5*b)/(512*c**4) + x*(b**10 - 5*b**8 + 10*b**6 - 10
*b**4 + 5*b**2 - 1)/(1024*c**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(85) = 170.
time = 0.65, size = 334, normalized size = 3.06

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^{9}x^{10} + 887040\,b^{2}c^{8}x^{9} + 1330560\,b^{3}c^{7}x^{8} + 1330560\,b^{4}c^{6}x^{7} - 98560\,c^{8}x^{9} + 931392\,b^{5}c^{5}x^{6} - 443520\,bc^{7}x^{8} + 465696\,b^{6}c^{4}x^{5} - 887040\,b^{2}c^{6}x^{7} + 166320\,b^{7}c^{3}x^{4} - 1034880\,b^{3}c^{5}x^{6} + 41580\,b^{8}c^{2}x^{3} - 776160\,b^{4}c^{4}x^{5} + 63360\,c^{6}x^{7} + 6930\,b^{9}cx^{2} - 388080\,b^{5}c^{3}x^{4} + 221760\,bc^{5}x^{6} + 693\,b^{10}x - 129360\,b^{6}c^{2}x^{3} + 332640\,b^{2}c^{4}x^{5} - 27720\,b^{7}cx^{2} + 277200\,b^{3}c^{3}x^{4} - 3465\,b^{8}x + 138600\,b^{4}c^{2}x^{3} - 22176\,c^{4}x^{5} + 41580\,b^{5}cx^{2} - 55440\,bc^{3}x^{4} + 6930\,b^{6}x - 55440\,b^{2}c^{2}x^{3} - 27720\,b^{3}cx^{2} - 6930\,b^{4}x + 4620\,c^{2}x^{3} + 6930\,bcx^{2} + 3465\,b^{2}x - 693\,x}{709632\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 98560*c^8*x^9 + 931392*b^5*c^5*x^6 -
443520*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 166320*b^7*c^3
*x^4 - 1034880*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 63360
*c^6*x^7 + 6930*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 221760*b*c^5*x^6 + 693*b^1
0*x - 129360*b^6*c^2*x^3 + 332640*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 277200*b^
3*c^3*x^4 - 3465*b^8*x + 138600*b^4*c^2*x^3 - 22176*c^4*x^5 + 41580*b^5*c*x
^2 - 55440*b*c^3*x^4 + 6930*b^6*x - 55440*b^2*c^2*x^3 - 27720*b^3*c*x^2 - 6
930*b^4*x + 4620*c^2*x^3 + 6930*b*c*x^2 + 3465*b^2*x - 693*x)/c^5
```

Mupad [B]
time = 0.31, size = 184, normalized size = 1.69

$$\frac{c^5\,x^{11}}{11}+\frac{x\,{\left(b^2-1\right)}^5}{1024\,c^5}+\frac{b\,x^6\,\left(63\,b^4-70\,b^2+15\right)}{48}+\frac{5\,c\,x^7\,\left(21\,b^4-14\,b^2+1\right)}{56}+\frac{b\,c^4\,x^{10}}{2}+\frac{5\,c^3\,x^9\,\left(9\,b^2-1\right)}{36}+\frac{x^5\,\left(21\,b^6-35\,b^4+15\,b^2-1\right)}{32\,c}+\frac{5\,b\,c^2\,x^8\,\left(3\,b^2-1\right)}{8}+\frac{5\,b\,x^2\,{\left(b^2-1\right)}^4}{512\,c^4}+\frac{5\,x^3\,{\left(b^2-1\right)}^3\,\left(9\,b^2-1\right)}{768\,c^3}+\frac{5\,b\,x^4\,{\left(b^2-1\right)}^2\,\left(3\,b^2-1\right)}{64\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + (b^2/4 - 1/4)/c)^5,x)
```

```
[Out] (c^5*x^11)/11 + (x*(b^2 - 1)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 70*b^2 + 15))
/48 + (5*c*x^7*(21*b^4 - 14*b^2 + 1))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9*(9*b
^2 - 1))/36 + (x^5*(15*b^2 - 35*b^4 + 21*b^6 - 1))/(32*c) + (5*b*c^2*x^8*(3
*b^2 - 1))/8 + (5*b*x^2*(b^2 - 1)^4)/(512*c^4) + (5*x^3*(b^2 - 1)^3*(9*b^2
- 1))/(768*c^3) + (5*b*x^4*(b^2 - 1)^2*(3*b^2 - 1))/(64*c^2)
```

3.75 $\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal. Leaf size=109

$$\frac{(2-b-2cx)^6}{12c^6}-\frac{5(2-b-2cx)^7}{56c^6}+\frac{5(2-b-2cx)^8}{128c^6}-\frac{5(2-b-2cx)^9}{576c^6}+\frac{(2-b-2cx)^{10}}{1024c^6}-\frac{(2-b-2cx)^{11}}{22528c^6}$$

```
[Out] 1/12*(-2*c*x-b+2)^6/c^6-5/56*(-2*c*x-b+2)^7/c^6+5/128*(-2*c*x-b+2)^8/c^6-5/
576*(-2*c*x-b+2)^9/c^6+1/1024*(-2*c*x-b+2)^10/c^6-1/22528*(-2*c*x-b+2)^11/c
^6
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {624, 45}

$$-\frac{(-b-2cx+2)^{11}}{22528c^6}+\frac{(-b-2cx+2)^{10}}{1024c^6}-\frac{5(-b-2cx+2)^9}{576c^6}+\frac{5(-b-2cx+2)^8}{128c^6}-\frac{5(-b-2cx+2)^7}{56c^6}+\frac{(-b-2cx+2)^6}{12c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (2 - b - 2*c*x)^6/(12*c^6) - (5*(2 - b - 2*c*x)^7)/(56*c^6) + (5*(2 - b - 2
*c*x)^8)/(128*c^6) - (5*(2 - b - 2*c*x)^9)/(576*c^6) + (2 - b - 2*c*x)^10/(
1024*c^6) - (2 - b - 2*c*x)^11/(22528*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 624

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-2+b)+cx\right)^5 \left(\frac{2+b}{2}+cx\right)^5 dx}{c^5}$$
$$= \frac{\int \left(32\left(\frac{1}{2}(-2+b)+cx\right)^5+80\left(\frac{1}{2}(-2+b)+cx\right)^6+80\left(\frac{1}{2}(-2+b)+cx\right)^7+\right.}{c^5}$$
$$= \frac{(2-b-2cx)^6}{12c^6}-\frac{5(2-b-2cx)^7}{56c^6}+\frac{5(2-b-2cx)^8}{128c^6}-\frac{5(2-b-2cx)^9}{576c^6}+$$

Mathematica [A]
time = 0.03, size = 207, normalized size = 1.90

$$\frac{(-4+b^2)^5 x}{1024c^5}+\frac{5b(-4+b^2)^4 x^2}{512c^4}+\frac{5(-4+b^2)^3(-4+9b^2)x^3}{768c^3}+\frac{5b(-4+b^2)^2(-4+3b^2)x^4}{64c^2}+\frac{(-4+b^2)(16-56b^2+21b^4)x^5}{32c}+\frac{1}{48}b(240-280b^2+63b^4)x^6+\frac{5}{56}(16-56b^2+21b^4)cx^7+\frac{5}{8}(-4b+3b^3)c^2x^8+\frac{5}{36}(-4+9b^2)c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-4 + b^2)^5*x)/(1024*c^5) + (5*b*(-4 + b^2)^4*x^2)/(512*c^4) + (5*(-4 + b
^2)^3*(-4 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-4 + b^2)^2*(-4 + 3*b^2)*x^4)/(64
*c^2) + ((-4 + b^2)*(16 - 56*b^2 + 21*b^4)*x^5)/(32*c) + (b*(240 - 280*b^2
+ 63*b^4)*x^6)/48 + (5*(16 - 56*b^2 + 21*b^4)*c*x^7)/56 + (5*(-4*b + 3*b^3)
*c^2*x^8)/8 + (5*(-4 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(97) = 194$.
time = 0.53, size = 636, normalized size = 5.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-4)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-4)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-2
)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-4)*c^2*b+b*(2*(3/2*b^2-2)*c^2+4*b^2*c^2)+c*
((b^2-4)*c*b+4*(3/2*b^2-2)*b*c))*x^8+1/7*(1/4*(b^2-4)/c*(2*(3/2*b^2-2)*c^2+
4*b^2*c^2)+b*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2
+(3/2*b^2-2)^2))*x^7+1/6*(1/4*(b^2-4)/c*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+b*(
1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2-2)^2)+c*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b
*(3/2*b^2-2)))*x^6+1/5*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2
-2)^2)+b*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2*b^2-2))+c*(1/8*(b^2-4)^2/c^2*(
3/2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-4)/c*(1/4*(b^2-4)^2/c*
b+(b^2-4)/c*b*(3/2*b^2-2))+b*(1/8*(b^2-4)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)^2/c
^2*b^2)+1/16/c^2*(b^2-4)^3*b)*x^4+1/3*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2/c^2*(3/
2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2)+1/16*b^2*(b^2-4)^3/c^3+1/256/c^3*(b^2-4)^4)
*x^3+5/512*(b^2-4)^4/c^4*b*x^2+1/1024*(b^2-4)^5/c^5*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.
time = 0.28, size = 234, normalized size = 2.15

$$\frac{1}{11}\,c^{5}x^{11}+\frac{1}{2}\,bc^{4}x^{10}+\frac{10}{9}\,b^{2}c^{3}x^{9}+\frac{5}{4}\,b^{3}c^{2}x^{8}+\frac{5}{7}\,b^{4}cx^{7}+\frac{1}{6}\,b^{5}x^{6}+\frac{5\,(2\,cx^{3}+3\,bx^{2})(b^{2}-4)^{4}}{1536\,c^{4}}+\frac{(6\,c^{2}x^{5}+15\,bcx^{4}+10\,b^{2}x^{3})(b^{2}-4)^{3}}{192\,c^{3}}+\frac{(20\,c^{3}x^{7}+70\,bc^{2}x^{6}+84\,b^{2}cx^{5}+35\,b^{3}x^{4})(b^{2}-4)^{2}}{224\,c^{2}}+\frac{(70\,c^{4}x^{9}+315\,bc^{3}x^{8}+540\,b^{2}c^{2}x^{7}+420\,b^{3}cx^{6}+126\,b^{4}x^{5})(b^{2}-4)}{504\,c}+\frac{(b^{2}-4)^{5}x}{1024\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 4)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 4)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 4)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 4)/c + 1/1024*(b^2 - 4)^5*x/c^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(85) = 170$.
time = 1.05, size = 235, normalized size = 2.16

$$\frac{64512\,c^{10}x^{11}+354816\,bc^{9}x^{10}+98560\,(9\,b^{2}-4)c^{8}x^{9}+443520\,(3\,b^{3}-4\,b)c^{7}x^{8}+63360\,(21\,b^{4}-56\,b^{2}+16)c^{6}x^{7}+14784\,(63\,b^{5}-280\,b^{3}+240\,b)c^{5}x^{6}+22176\,(21\,b^{6}-140\,b^{4}+240\,b^{2}-64)c^{4}x^{5}+55440\,(3\,b^{7}-28\,b^{5}+80\,b^{3}-64\,b)c^{3}x^{4}+4620\,(9\,b^{8}-112\,b^{6}+480\,b^{4}-768\,b^{2}+256)c^{2}x^{3}+6930\,(b^{9}-16\,b^{7}+96\,b^{5}-256\,b^{3}+256\,b)cx^{2}+693\,(b^{10}-20\,b^{8}+160\,b^{6}-640\,b^{4}+1280\,b^{2}-1024)x}{709632\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 4)*c^8*x^9 +
 443520*(3*b^3 - 4*b)*c^7*x^8 + 63360*(21*b^4 - 56*b^2 + 16)*c^6*x^7 + 1478
4*(63*b^5 - 280*b^3 + 240*b)*c^5*x^6 + 22176*(21*b^6 - 140*b^4 + 240*b^2 -
64)*c^4*x^5 + 55440*(3*b^7 - 28*b^5 + 80*b^3 - 64*b)*c^3*x^4 + 4620*(9*b^8
- 112*b^6 + 480*b^4 - 768*b^2 + 256)*c^2*x^3 + 6930*(b^9 - 16*b^7 + 96*b^5
- 256*b^3 + 256*b)*c*x^2 + 693*(b^10 - 20*b^8 + 160*b^6 - 640*b^4 + 1280*b^
2 - 1024)*x)/c^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.
time = 0.07, size = 250, normalized size = 2.29

$$\frac{bc^{4}x^{10}}{2}+\frac{c^{5}x^{11}}{11}+x^{9}\cdot\left(\frac{5b^{2}c^{3}}{4}-\frac{5c^{3}}{9}\right)+x^{8}\cdot\left(\frac{15b^{3}c^{2}}{8}-\frac{5bc^{2}}{2}\right)+x^{7}\cdot\left(\frac{15b^{4}c}{8}-5b^{2}c+\frac{10c}{7}\right)+x^{6}\cdot\left(\frac{21b^{5}}{16}-\frac{35b^{3}}{6}+5b\right)+\frac{x^{5}\cdot(21b^{6}-140b^{4}+240b^{2}-64)}{32c}+\frac{x^{4}\cdot(15b^{7}-140b^{5}+400b^{3}-320b)}{64c^{2}}+\frac{x^{3}\cdot(45b^{8}-560b^{6}+2400b^{4}-3840b^{2}+1280)}{768c^{3}}+\frac{x^{2}\cdot(5b^{9}-80b^{7}+480b^{5}-1280b^{3}+1280b)}{512c^{4}}+\frac{x(b^{10}-20b^{8}+160b^{6}-640b^{4}+1280b^{2}-1024)}{1024c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-4)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/9) + x**8*(15
*b**3*c**2/8 - 5*b*c**2/2) + x**7*(15*b**4*c/8 - 5*b**2*c + 10*c/7) + x**6*
(21*b**5/16 - 35*b**3/6 + 5*b) + x**5*(21*b**6 - 140*b**4 + 240*b**2 - 64)/
```

```
(32*c) + x**4*(15*b**7 - 140*b**5 + 400*b**3 - 320*b)/(64*c**2) + x**3*(45*
b**8 - 560*b**6 + 2400*b**4 - 3840*b**2 + 1280)/(768*c**3) + x**2*(5*b**9 -
 80*b**7 + 480*b**5 - 1280*b**3 + 1280*b)/(512*c**4) + x*(b**10 - 20*b**8 +
 160*b**6 - 640*b**4 + 1280*b**2 - 1024)/(1024*c**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(85) = 170.
time = 1.37, size = 334, normalized size = 3.06

$$\frac{64512\,c^{10}x^{11}+354816\,bc^{9}x^{10}+887040\,b^{2}c^{8}x^{9}+1330560\,b^{3}c^{7}x^{8}+1330560\,b^{4}c^{6}x^{7}-394240\,c^{8}x^{9}+931392\,b^{5}c^{5}x^{6}-1774080\,bc^{7}x^{8}+465696\,b^{6}c^{4}x^{5}-3548160\,b^{2}c^{6}x^{7}+166320\,b^{7}c^{3}x^{4}-4139520\,b^{3}c^{5}x^{6}+41580\,b^{8}c^{2}x^{3}-3104640\,b^{4}c^{4}x^{5}+1013760\,c^{6}x^{7}+6930\,b^{9}cx^{2}-1552320\,b^{5}c^{3}x^{4}+3548160\,bc^{5}x^{6}+693\,b^{10}x-517440\,b^{6}c^{2}x^{3}+5322240\,b^{2}c^{4}x^{5}-110880\,b^{7}cx^{2}+4435200\,b^{3}c^{3}x^{4}-13860\,b^{8}x+2217600\,b^{4}c^{2}x^{3}-1419264\,c^{4}x^{5}+665280\,b^{5}cx^{2}-3548160\,bc^{3}x^{4}+110880\,b^{6}x-3548160\,b^{2}c^{2}x^{3}-1774080\,b^{3}cx^{2}-443520\,b^{4}x+1182720\,c^{2}x^{3}+1774080\,bcx^{2}+887040\,b^{2}x-709632\,x}{709632\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 394240*c^8*x^9 + 931392*b^5*c^5*x^6 -
 1774080*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 3548160*b^2*c^6*x^7 + 166320*b^7*
c^3*x^4 - 4139520*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 3104640*b^4*c^4*x^5 + 1
013760*c^6*x^7 + 6930*b^9*c*x^2 - 1552320*b^5*c^3*x^4 + 3548160*b*c^5*x^6 +
 693*b^10*x - 517440*b^6*c^2*x^3 + 5322240*b^2*c^4*x^5 - 110880*b^7*c*x^2 +
 4435200*b^3*c^3*x^4 - 13860*b^8*x + 2217600*b^4*c^2*x^3 - 1419264*c^4*x^5
+ 665280*b^5*c*x^2 - 3548160*b*c^3*x^4 + 110880*b^6*x - 3548160*b^2*c^2*x^3
 - 1774080*b^3*c*x^2 - 443520*b^4*x + 1182720*c^2*x^3 + 1774080*b*c*x^2 + 8
87040*b^2*x - 709632*x)/c^5
```

Mupad [B]
time = 0.33, size = 184, normalized size = 1.69

$$\frac{c^5\,x^{11}}{11}+\frac{x\,{\left(b^2-4\right)}^5}{1024\,c^5}+\frac{b\,x^6\,\left(63\,b^4-280\,b^2+240\right)}{48}+\frac{5\,c\,x^7\,\left(21\,b^4-56\,b^2+16\right)}{56}+\frac{b\,c^4\,x^{10}}{2}+\frac{5\,c^3\,x^9\,\left(9\,b^2-4\right)}{36}+\frac{x^5\,\left(21\,b^6-140\,b^4+240\,b^2-64\right)}{32\,c}+\frac{5\,b\,c^2\,x^8\,\left(3\,b^2-4\right)}{8}+\frac{5\,b\,x^2\,{\left(b^2-4\right)}^4}{512\,c^4}+\frac{5\,x^3\,{\left(b^2-4\right)}^3\,\left(9\,b^2-4\right)}{768\,c^3}+\frac{5\,b\,x^4\,{\left(b^2-4\right)}^2\,\left(3\,b^2-4\right)}{64\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + (b^2/4 - 1)/c)^5,x)
```

```
[Out] (c^5*x^11)/11 + (x*(b^2 - 4)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 280*b^2 + 240
))/48 + (5*c*x^7*(21*b^4 - 56*b^2 + 16))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9*(
9*b^2 - 4))/36 + (x^5*(240*b^2 - 140*b^4 + 21*b^6 - 64))/(32*c) + (5*b*c^2*
x^8*(3*b^2 - 4))/8 + (5*b*x^2*(b^2 - 4)^4)/(512*c^4) + (5*x^3*(b^2 - 4)^3*(
9*b^2 - 4))/(768*c^3) + (5*b*x^4*(b^2 - 4)^2*(3*b^2 - 4))/(64*c^2)
```

3.76 $\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal. Leaf size=109

$$\frac{81(3-b-2cx)^6}{128c^6}-\frac{405(3-b-2cx)^7}{896c^6}+\frac{135(3-b-2cx)^8}{1024c^6}-\frac{5(3-b-2cx)^9}{256c^6}+\frac{3(3-b-2cx)^{10}}{2048c^6}-\frac{(3-b-2c}{22528c}$$

```
[Out] 81/128*(-2*c*x-b+3)^6/c^6-405/896*(-2*c*x-b+3)^7/c^6+135/1024*(-2*c*x-b+3)^
8/c^6-5/256*(-2*c*x-b+3)^9/c^6+3/2048*(-2*c*x-b+3)^10/c^6-1/22528*(-2*c*x-b
+3)^11/c^6
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {624, 45}

$$-\frac{(-b-2cx+3)^{11}}{22528c^6}+\frac{3(-b-2cx+3)^{10}}{2048c^6}-\frac{5(-b-2cx+3)^9}{256c^6}+\frac{135(-b-2cx+3)^8}{1024c^6}-\frac{405(-b-2cx+3)^7}{896c^6}+\frac{81(-b-2cx+3)^6}{128c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (81*(3 - b - 2*c*x)^6)/(128*c^6) - (405*(3 - b - 2*c*x)^7)/(896*c^6) + (135
*(3 - b - 2*c*x)^8)/(1024*c^6) - (5*(3 - b - 2*c*x)^9)/(256*c^6) + (3*(3 -
b - 2*c*x)^10)/(2048*c^6) - (3 - b - 2*c*x)^11/(22528*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 624

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \left(\frac{-9+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-3+b)+cx\right)^5 \left(\frac{3+b}{2}+cx\right)^5 dx}{c^5}$$

$$= \frac{\int \left(243\left(\frac{1}{2}(-3+b)+cx\right)^5+405\left(\frac{1}{2}(-3+b)+cx\right)^6+270\left(\frac{1}{2}(-3+b)+cx\right)^7\right.}{c}$$

$$= \frac{81(3-b-2cx)^6}{128c^6}-\frac{405(3-b-2cx)^7}{896c^6}+\frac{135(3-b-2cx)^8}{1024c^6}-\frac{5(3-b-2c}{256c^6}$$

Mathematica [A]
time = 0.02, size = 199, normalized size = 1.83

$$\frac{(-9+b^2)^5 x}{1024c^5}+\frac{5b(-9+b^2)^4 x^2}{512c^4}+\frac{15(-9+b^2)^3 (-1+b^2) x^3}{256c^3}+\frac{15b(-9+b^2)^2 (-3+b^2) x^4}{64c^2}+\frac{3(-9+b^2)(27-42b^2+7b^4) x^5}{32c}+\frac{3}{16}b(135-70b^2+7b^4) x^6+\frac{15}{56}(27-42b^2+7b^4) cx^7+\frac{15}{8}(-3b+b^3) c^2x^8+\frac{5}{4}(-1+b^2) c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-9 + b^2)^5*x)/(1024*c^5) + (5*b*(-9 + b^2)^4*x^2)/(512*c^4) + (15*(-9 +
b^2)^3*(-1 + b^2)*x^3)/(256*c^3) + (15*b*(-9 + b^2)^2*(-3 + b^2)*x^4)/(64*c
^2) + (3*(-9 + b^2)*(27 - 42*b^2 + 7*b^4)*x^5)/(32*c) + (3*b*(135 - 70*b^2
+ 7*b^4)*x^6)/16 + (15*(27 - 42*b^2 + 7*b^4)*c*x^7)/56 + (15*(-3*b + b^3)*c
^2*x^8)/8 + (5*(-1 + b^2)*c^3*x^9)/4 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(97) = 194$.
time = 0.49, size = 636, normalized size = 5.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-9)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-9)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-9
/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-9)*c^2*b+b*(2*(3/2*b^2-9/2)*c^2+4*b^2*c^2
)+c*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c))*x^8+1/7*(1/4*(b^2-9)/c*(2*(3/2*b^2-9
/2)*c^2+4*b^2*c^2)+b*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c)+c*(1/8*(b^2-9)^2+2*(
b^2-9)*b^2+(3/2*b^2-9/2)^2))*x^7+1/6*(1/4*(b^2-9)/c*((b^2-9)*c*b+4*(3/2*b^2
-9/2)*b*c)+b*(1/8*(b^2-9)^2+2*(b^2-9)*b^2+(3/2*b^2-9/2)^2)+c*(1/4*(b^2-9)^2
/c*b+(b^2-9)/c*b*(3/2*b^2-9/2)))*x^6+1/5*(1/4*(b^2-9)/c*(1/8*(b^2-9)^2+2*(b
^2-9)*b^2+(3/2*b^2-9/2)^2)+b*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2))+
c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2))*x^5+1/4*(1/4*(b^
2-9)/c*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2))+b*(1/8*(b^2-9)^2/c^2*(
3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16/c^2*(b^2-9)^3*b)*x^4+1/3*(1/4*(b^2
-9)/c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16*b^2*(b^2
-9)^3/c^3+1/256/c^3*(b^2-9)^4)*x^3+5/512*(b^2-9)^4/c^4*b*x^2+1/1024*(b^2-9)
^5/c^5*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.
time = 0.29, size = 234, normalized size = 2.15

$$\frac{1}{11}c^5x^{11} + \frac{1}{2}bc^4x^{10} + \frac{10}{9}b^2c^3x^9 + \frac{5}{4}b^3c^2x^8 + \frac{5}{7}b^4cx^7 + \frac{1}{6}b^5x^6 + \frac{5\,(2\,cx^3 + 3\,bx^2)(b^2 - 9)^4}{1536\,c^4} + \frac{(6\,c^2x^5 + 15\,bcx^4 + 10\,b^2x^3)(b^2 - 9)^3}{192\,c^3} + \frac{(20\,c^3x^7 + 70\,bc^2x^6 + 84\,b^2cx^5 + 35\,b^3x^4)(b^2 - 9)^2}{224\,c^2} + \frac{(70\,c^4x^9 + 315\,bc^3x^8 + 540\,b^2c^2x^7 + 420\,b^3cx^6 + 126\,b^4x^5)(b^2 - 9)}{504\,c} + \frac{(b^2 - 9)^5x}{1024\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 9)^4/c^4 + 1/192
*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 9)^3/c^3 + 1/224*(20*c^3*x^7
+ 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 9)^2/c^2 + 1/504*(70*c^4
*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2
- 9)/c + 1/1024*(b^2 - 9)^5*x/c^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(85) = 170$.
time = 1.77, size = 227, normalized size = 2.08

$$\frac{7168\,c^{10}x^{11} + 39424\,bc^9x^{10} + 98560\,(b^2 - 1)c^8x^9 + 147840\,(b^3 - 3\,b)c^7x^8 + 21120\,(7\,b^4 - 42\,b^2 + 27)c^6x^7 + 14784\,(7\,b^5 - 70\,b^3 + 135\,b)c^5x^6 + 7392\,(7\,b^6 - 105\,b^4 + 405\,b^2 - 243)c^4x^5 + 18480\,(b^7 - 21\,b^5 + 135\,b^3 - 243\,b)c^3x^4 + 4620\,(b^8 - 28\,b^6 + 270\,b^4 - 972\,b^2 + 729)c^2x^3 + 770\,(b^9 - 36\,b^7 + 486\,b^5 - 2916\,b^3 + 6561\,b)cx^2 + 77\,(b^{10} - 45\,b^8 + 810\,b^6 - 7290\,b^4 + 32805\,b^2 - 59049)x}{78848\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*(b^2 - 1)*c^8*x^9 + 1478
40*(b^3 - 3*b)*c^7*x^8 + 21120*(7*b^4 - 42*b^2 + 27)*c^6*x^7 + 14784*(7*b^5
 - 70*b^3 + 135*b)*c^5*x^6 + 7392*(7*b^6 - 105*b^4 + 405*b^2 - 243)*c^4*x^5
 + 18480*(b^7 - 21*b^5 + 135*b^3 - 243*b)*c^3*x^4 + 4620*(b^8 - 28*b^6 + 27
0*b^4 - 972*b^2 + 729)*c^2*x^3 + 770*(b^9 - 36*b^7 + 486*b^5 - 2916*b^3 + 6
561*b)*c*x^2 + 77*(b^10 - 45*b^8 + 810*b^6 - 7290*b^4 + 32805*b^2 - 59049)*
x)/c^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(99) = 198$.
time = 0.07, size = 253, normalized size = 2.32

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\cdot\left(\frac{5b^2c^3}{4} - \frac{5c^3}{4}\right) + x^8\cdot\left(\frac{15b^3c^2}{8} - \frac{45bc^2}{8}\right) + x^7\cdot\left(\frac{15b^4c}{8} - \frac{45b^2c}{4} + \frac{405c}{56}\right) + x^6\cdot\left(\frac{21b^5}{16} - \frac{105b^3}{8} + \frac{405b}{16}\right) + \frac{x^5\cdot(21b^6 - 315b^4 + 1215b^2 - 729)}{32c} + \frac{x^4\cdot(15b^7 - 315b^5 + 2025b^3 - 3645b)}{64c^2} + \frac{x^3\cdot(15b^8 - 420b^6 + 4050b^4 - 14580b^2 + 10935)}{256c^3} + \frac{x^2\cdot(5b^9 - 180b^7 + 2430b^5 - 14580b^3 + 32805b)}{512c^4} + \frac{x(b^{10} - 45b^8 + 810b^6 - 7290b^4 + 32805b^2 - 59049)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-9)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/4) + x**8*(15
*b**3*c**2/8 - 45*b*c**2/8) + x**7*(15*b**4*c/8 - 45*b**2*c/4 + 405*c/56) +
 x**6*(21*b**5/16 - 105*b**3/8 + 405*b/16) + x**5*(21*b**6 - 315*b**4 + 121
```

```
5*b**2 - 729)/(32*c) + x**4*(15*b**7 - 315*b**5 + 2025*b**3 - 3645*b)/(64*c
**2) + x**3*(15*b**8 - 420*b**6 + 4050*b**4 - 14580*b**2 + 10935)/(256*c**3
) + x**2*(5*b**9 - 180*b**7 + 2430*b**5 - 14580*b**3 + 32805*b)/(512*c**4)
+ x*(b**10 - 45*b**8 + 810*b**6 - 7290*b**4 + 32805*b**2 - 59049)/(1024*c**
5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(85) = 170$.
time = 1.79, size = 334, normalized size = 3.06

$$\frac{7168\,c^{10}x^{11}+39424\,bc^{9}x^{10}+98560\,b^{2}c^{8}x^{9}+147840\,b^{3}c^{7}x^{8}+147840\,b^{4}c^{6}x^{7}-98560\,c^{8}x^{9}+103488\,b^{5}c^{5}x^{6}-443520\,bc^{7}x^{8}+51744\,b^{6}c^{4}x^{5}-887040\,b^{2}c^{6}x^{7}+18480\,b^{7}c^{3}x^{4}-1034880\,b^{3}c^{5}x^{6}+4620\,b^{8}c^{2}x^{3}-776160\,b^{4}c^{4}x^{5}+570240\,c^{6}x^{7}+770\,b^{9}cx^{2}-388080\,b^{5}c^{3}x^{4}+1995840\,bc^{5}x^{6}+77\,b^{10}x-129360\,b^{6}c^{2}x^{3}+2993760\,b^{2}c^{4}x^{5}-27720\,b^{7}cx^{2}+2494800\,b^{3}c^{3}x^{4}-3465\,b^{8}x+1247400\,b^{4}c^{2}x^{3}-1796256\,c^{4}x^{5}+374220\,b^{5}cx^{2}-4490640\,bc^{3}x^{4}+62370\,b^{6}x-4490640\,b^{2}c^{2}x^{3}-2245320\,b^{3}cx^{2}-561330\,b^{4}x+3367980\,c^{2}x^{3}+5051970\,bcx^{2}+2525985\,b^{2}x-4546773\,x}{78848\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*b^2*c^8*x^9 + 147840*b^3
*c^7*x^8 + 147840*b^4*c^6*x^7 - 98560*c^8*x^9 + 103488*b^5*c^5*x^6 - 443520
*b*c^7*x^8 + 51744*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 18480*b^7*c^3*x^4 - 1
034880*b^3*c^5*x^6 + 4620*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 570240*c^6*x^7
 + 770*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 1995840*b*c^5*x^6 + 77*b^10*x - 129
360*b^6*c^2*x^3 + 2993760*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 2494800*b^3*c^3*x
^4 - 3465*b^8*x + 1247400*b^4*c^2*x^3 - 1796256*c^4*x^5 + 374220*b^5*c*x^2
- 4490640*b*c^3*x^4 + 62370*b^6*x - 4490640*b^2*c^2*x^3 - 2245320*b^3*c*x^2
 - 561330*b^4*x + 3367980*c^2*x^3 + 5051970*b*c*x^2 + 2525985*b^2*x - 45467
73*x)/c^5
```

Mupad [B]
time = 0.34, size = 176, normalized size = 1.61

$$\frac{c^5\,x^{11}}{11}+\frac{5\,c^3\,x^9\,\left(b^2-1\right)}{4}+\frac{x\,{\left(b^2-9\right)}^5}{1024\,c^5}+\frac{3\,b\,x^6\,\left(7\,b^4-70\,b^2+135\right)}{16}+\frac{15\,c\,x^7\,\left(7\,b^4-42\,b^2+27\right)}{56}+\frac{b\,c^4\,x^{10}}{2}+\frac{3\,x^5\,\left(7\,b^6-105\,b^4+405\,b^2-243\right)}{32\,c}+\frac{15\,b\,c^2\,x^8\,\left(b^2-3\right)}{8}+\frac{15\,x^3\,\left(b^2-1\right)\,{\left(b^2-9\right)}^3}{256\,c^3}+\frac{5\,b\,x^2\,{\left(b^2-9\right)}^4}{512\,c^4}+\frac{15\,b\,x^4\,\left(b^2-3\right)\,{\left(b^2-9\right)}^2}{64\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + (b^2/4 - 9/4)/c)^5,x)
```

```
[Out] (c^5*x^11)/11 + (5*c^3*x^9*(b^2 - 1))/4 + (x*(b^2 - 9)^5)/(1024*c^5) + (3*b
*x^6*(7*b^4 - 70*b^2 + 135))/16 + (15*c*x^7*(7*b^4 - 42*b^2 + 27))/56 + (b*
c^4*x^10)/2 + (3*x^5*(405*b^2 - 105*b^4 + 7*b^6 - 243))/(32*c) + (15*b*c^2*
x^8*(b^2 - 3))/8 + (15*x^3*(b^2 - 1)*(b^2 - 9)^3)/(256*c^3) + (5*b*x^2*(b^2
 - 9)^4)/(512*c^4) + (15*b*x^4*(b^2 - 3)*(b^2 - 9)^2)/(64*c^2)
```

3.77 $\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal. Leaf size=109

$$\frac{8(4-b-2cx)^6}{3c^6}-\frac{10(4-b-2cx)^7}{7c^6}+\frac{5(4-b-2cx)^8}{16c^6}-\frac{5(4-b-2cx)^9}{144c^6}+\frac{(4-b-2cx)^{10}}{512c^6}-\frac{(4-b-2cx)^{11}}{22528c^6}$$

```
[Out] 8/3*(-2*c*x-b+4)^6/c^6-10/7*(-2*c*x-b+4)^7/c^6+5/16*(-2*c*x-b+4)^8/c^6-5/14
4*(-2*c*x-b+4)^9/c^6+1/512*(-2*c*x-b+4)^10/c^6-1/22528*(-2*c*x-b+4)^11/c^6
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {624, 45}

$$-\frac{(-b-2cx+4)^{11}}{22528c^6}+\frac{(-b-2cx+4)^{10}}{512c^6}-\frac{5(-b-2cx+4)^9}{144c^6}+\frac{5(-b-2cx+4)^8}{16c^6}-\frac{10(-b-2cx+4)^7}{7c^6}+\frac{8(-b-2cx+4)^6}{3c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (8*(4 - b - 2*c*x)^6)/(3*c^6) - (10*(4 - b - 2*c*x)^7)/(7*c^6) + (5*(4 - b
- 2*c*x)^8)/(16*c^6) - (5*(4 - b - 2*c*x)^9)/(144*c^6) + (4 - b - 2*c*x)^10
/(512*c^6) - (4 - b - 2*c*x)^11/(22528*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 624

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c
*x, x]^p, x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 dx = \frac{\int \left(\frac{1}{2}(-4+b)+cx\right)^5 \left(\frac{4+b}{2}+cx\right)^5 dx}{c^5}$$

$$= \frac{\int \left(1024\left(\frac{1}{2}(-4+b)+cx\right)^5+1280\left(\frac{1}{2}(-4+b)+cx\right)^6+640\left(\frac{1}{2}(-4+b)+\right.\right.}{}$$

$$= \frac{8(4-b-2cx)^6}{3c^6}-\frac{10(4-b-2cx)^7}{7c^6}+\frac{5(4-b-2cx)^8}{16c^6}-\frac{5(4-b-2cx)^9}{144c^6}$$

Mathematica [A]
time = 0.03, size = 207, normalized size = 1.90

$$\frac{(-16+b^2)^5 x}{1024c^5}+\frac{5b(-16+b^2)^4 x^2}{512c^4}+\frac{5(-16+b^2)^3(-16+9b^2) x^3}{768c^3}+\frac{5b(-16+b^2)^2(-16+3b^2) x^4}{64c^2}+\frac{(-16+b^2)(256-224b^2+21b^4) x^5}{32c}+\frac{1}{48}b(3840-1120b^2+63b^4) x^6+\frac{5}{56}(256-224b^2+21b^4) cx^7+\frac{5}{8}(-16b+3b^3) c^2x^8+\frac{5}{36}(-16+9b^2) c^3x^9+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-16 + b^2)^5*x)/(1024*c^5) + (5*b*(-16 + b^2)^4*x^2)/(512*c^4) + (5*(-16
+ b^2)^3*(-16 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-16 + b^2)^2*(-16 + 3*b^2)*x^
4)/(64*c^2) + ((-16 + b^2)*(256 - 224*b^2 + 21*b^4)*x^5)/(32*c) + (b*(3840
- 1120*b^2 + 63*b^4)*x^6)/48 + (5*(256 - 224*b^2 + 21*b^4)*c*x^7)/56 + (5*(
-16*b + 3*b^3)*c^2*x^8)/8 + (5*(-16 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 +
 (c^5*x^11)/11
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(97) = 194$.
time = 0.48, size = 636, normalized size = 5.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*(b^2-16)/c+b*x+c*x^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-16)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-
8)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-16)*c^2*b+b*(2*(3/2*b^2-8)*c^2+4*b^2*c^2)+
c*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c))*x^8+1/7*(1/4*(b^2-16)/c*(2*(3/2*b^2-8)*
c^2+4*b^2*c^2)+b*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c)+c*(1/8*(b^2-16)^2+2*(b^2-
16)*b^2+(3/2*b^2-8)^2))*x^7+1/6*(1/4*(b^2-16)/c*((b^2-16)*c*b+4*(3/2*b^2-8)
*b*c)+b*(1/8*(b^2-16)^2+2*(b^2-16)*b^2+(3/2*b^2-8)^2)+c*(1/4*(b^2-16)^2/c*b
+(b^2-16)/c*b*(3/2*b^2-8)))*x^6+1/5*(1/4*(b^2-16)/c*(1/8*(b^2-16)^2+2*(b^2-
16)*b^2+(3/2*b^2-8)^2)+b*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8))+c*(1
/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-16
)/c*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8))+b*(1/8*(b^2-16)^2/c^2*(3/
2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16/c^2*(b^2-16)^3*b)*x^4+1/3*(1/4*(b^2-1
6)/c*(1/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16*b^2*(b^2-
```

```
16)^3/c^3+1/256/c^3*(b^2-16)^4)*x^3+5/512*(b^2-16)^4/c^4*b*x^2+1/1024*(b^2-
16)^5/c^5*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(85) = 170$.
time = 0.28, size = 234, normalized size = 2.15

$$\frac{1}{11}\,c^{5}x^{11}+\frac{1}{2}\,bc^{4}x^{10}+\frac{10}{9}\,b^{2}c^{3}x^{9}+\frac{5}{4}\,b^{3}c^{2}x^{8}+\frac{5}{7}\,b^{4}cx^{7}+\frac{1}{6}\,b^{5}x^{6}+\frac{5\,(2\,cx^{3}+3\,bx^{2})(b^{2}-16)^{4}}{1536\,c^{4}}+\frac{(6\,c^{2}x^{5}+15\,bcx^{4}+10\,b^{2}x^{3})(b^{2}-16)^{3}}{192\,c^{3}}+\frac{(20\,c^{3}x^{7}+70\,bc^{2}x^{6}+84\,b^{2}cx^{5}+35\,b^{3}x^{4})(b^{2}-16)^{2}}{224\,c^{2}}+\frac{(70\,c^{4}x^{9}+315\,bc^{3}x^{8}+540\,b^{2}c^{2}x^{7}+420\,b^{3}cx^{6}+126\,b^{4}x^{5})(b^{2}-16)}{504\,c}+\frac{(b^{2}-16)^{5}x}{1024\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b
^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 16)^4/c^4 + 1/19
2*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 16)^3/c^3 + 1/224*(20*c^3*x^
7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 16)^2/c^2 + 1/504*(70*
c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b
^2 - 16)/c + 1/1024*(b^2 - 16)^5*x/c^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(85) = 170$.
time = 1.12, size = 235, normalized size = 2.16

$$\frac{64512\,c^{10}x^{11}+354816\,bc^{9}x^{10}+98560\,(9\,b^{2}-16)c^{8}x^{9}+443520\,(3\,b^{3}-16\,b)c^{7}x^{8}+63360\,(21\,b^{4}-224\,b^{2}+256)c^{6}x^{7}+14784\,(63\,b^{5}-1120\,b^{3}+3840\,b)c^{5}x^{6}+22176\,(21\,b^{6}-560\,b^{4}+3840\,b^{2}-4096)c^{4}x^{5}+55440\,(3\,b^{7}-112\,b^{5}+1280\,b^{3}-4096\,b)c^{3}x^{4}+4620\,(9\,b^{8}-448\,b^{6}+7680\,b^{4}-49152\,b^{2}+65536)c^{2}x^{3}+6930\,(b^{9}-64\,b^{7}+1536\,b^{5}-16384\,b^{3}+65536\,b)cx^{2}+693\,(b^{10}-80\,b^{8}+2560\,b^{6}-40960\,b^{4}+327680\,b^{2}-1048576)x}{709632\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 16)*c^8*x^9
+ 443520*(3*b^3 - 16*b)*c^7*x^8 + 63360*(21*b^4 - 224*b^2 + 256)*c^6*x^7 +
14784*(63*b^5 - 1120*b^3 + 3840*b)*c^5*x^6 + 22176*(21*b^6 - 560*b^4 + 3840
*b^2 - 4096)*c^4*x^5 + 55440*(3*b^7 - 112*b^5 + 1280*b^3 - 4096*b)*c^3*x^4
+ 4620*(9*b^8 - 448*b^6 + 7680*b^4 - 49152*b^2 + 65536)*c^2*x^3 + 6930*(b^9
 - 64*b^7 + 1536*b^5 - 16384*b^3 + 65536*b)*c*x^2 + 693*(b^10 - 80*b^8 + 25
60*b^6 - 40960*b^4 + 327680*b^2 - 1048576)*x)/c^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(97) = 194$.
time = 0.07, size = 248, normalized size = 2.28

$$\frac{bc^{4}x^{10}}{2}+\frac{c^{5}x^{11}}{11}+x^{9}\cdot\left(\frac{5b^{2}c^{3}}{4}-\frac{20c^{3}}{9}\right)+x^{8}\cdot\left(\frac{15b^{3}c^{2}}{8}-10bc^{2}\right)+x^{7}\cdot\left(\frac{15b^{4}c}{8}-20b^{2}c+\frac{160c}{7}\right)+x^{6}\cdot\left(\frac{21b^{5}}{16}-\frac{70b^{3}}{3}+80b\right)+\frac{x^{5}\cdot(21b^{6}-560b^{4}+3840b^{2}-4096)}{32c}+\frac{x^{4}\cdot(15b^{7}-560b^{5}+6400b^{3}-20480b)}{64c^{2}}+\frac{x^{3}\cdot(45b^{8}-2240b^{6}+38400b^{4}-245760b^{2}+327680)}{768c^{3}}+\frac{x^{2}\cdot(5b^{9}-320b^{7}+7680b^{5}-81920b^{3}+327680b)}{512c^{4}}+\frac{x(b^{10}-80b^{8}+2560b^{6}-40960b^{4}+327680b^{2}-1048576)}{1024c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b**2-16)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 20*c**3/9) + x**8*(1
5*b**3*c**2/8 - 10*b*c**2) + x**7*(15*b**4*c/8 - 20*b**2*c + 160*c/7) + x**
```

```
6*(21*b**5/16 - 70*b**3/3 + 80*b) + x**5*(21*b**6 - 560*b**4 + 3840*b**2 -
4096)/(32*c) + x**4*(15*b**7 - 560*b**5 + 6400*b**3 - 20480*b)/(64*c**2) +
x**3*(45*b**8 - 2240*b**6 + 38400*b**4 - 245760*b**2 + 327680)/(768*c**3) +
 x**2*(5*b**9 - 320*b**7 + 7680*b**5 - 81920*b**3 + 327680*b)/(512*c**4) +
x*(b**10 - 80*b**8 + 2560*b**6 - 40960*b**4 + 327680*b**2 - 1048576)/(1024*
c**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(85) = 170.
time = 1.33, size = 334, normalized size = 3.06

$$\frac{64512\,c^{10}x^{11} + 354816\,bc^{9}x^{10} + 887040\,b^{2}c^{8}x^{9} + 1330560\,b^{3}c^{7}x^{8} + 1330560\,b^{4}c^{6}x^{7} - 1576960\,c^{8}x^{9} + 931392\,b^{5}c^{5}x^{6} - 7096320\,bc^{7}x^{8} + 465696\,b^{6}c^{4}x^{5} - 14192640\,b^{2}c^{6}x^{7} + 166320\,b^{7}c^{3}x^{4} - 16558080\,b^{3}c^{5}x^{6} + 41580\,b^{8}c^{2}x^{3} - 12418560\,b^{4}c^{4}x^{5} + 16220160\,c^{6}x^{7} + 6930\,b^{9}cx^{2} - 6209280\,b^{5}c^{3}x^{4} + 56770560\,bc^{5}x^{6} + 693\,b^{10}x - 2069760\,b^{6}c^{2}x^{3} + 85155840\,b^{2}c^{4}x^{5} - 443520\,b^{7}cx^{2} + 70963200\,b^{3}c^{3}x^{4} - 55440\,b^{8}x + 35481600\,b^{4}c^{2}x^{3} - 90832896\,c^{4}x^{5} + 10644480\,b^{5}cx^{2} - 227082240\,bc^{3}x^{4} + 1774080\,b^{6}x - 227082240\,b^{2}c^{2}x^{3} - 113541120\,b^{3}cx^{2} - 28385280\,b^{4}x + 302776320\,c^{2}x^{3} + 454164480\,bcx^{2} + 227082240\,b^{2}x - 726663168\,x}{709632\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 133056
0*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 1576960*c^8*x^9 + 931392*b^5*c^5*x^6
- 7096320*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 14192640*b^2*c^6*x^7 + 166320*b^
7*c^3*x^4 - 16558080*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 12418560*b^4*c^4*x^5
 + 16220160*c^6*x^7 + 6930*b^9*c*x^2 - 6209280*b^5*c^3*x^4 + 56770560*b*c^5
*x^6 + 693*b^10*x - 2069760*b^6*c^2*x^3 + 85155840*b^2*c^4*x^5 - 443520*b^7
*c*x^2 + 70963200*b^3*c^3*x^4 - 55440*b^8*x + 35481600*b^4*c^2*x^3 - 908328
96*c^4*x^5 + 10644480*b^5*c*x^2 - 227082240*b*c^3*x^4 + 1774080*b^6*x - 227
082240*b^2*c^2*x^3 - 113541120*b^3*c*x^2 - 28385280*b^4*x + 302776320*c^2*x
^3 + 454164480*b*c*x^2 + 227082240*b^2*x - 726663168*x)/c^5
```

Mupad [B]
time = 0.32, size = 184, normalized size = 1.69

$$\frac{c^5\,x^{11}}{11}+\frac{x\,{\left(b^2-16\right)}^5}{1024\,c^5}+\frac{b\,x^6\,\left(63\,b^4-1120\,b^2+3840\right)}{48}+\frac{5\,c\,x^7\,\left(21\,b^4-224\,b^2+256\right)}{56}+\frac{b\,c^4\,x^{10}}{2}+\frac{5\,c^3\,x^9\,\left(9\,b^2-16\right)}{36}+\frac{x^5\,\left(21\,b^6-560\,b^4+3840\,b^2-4096\right)}{32\,c}+\frac{5\,b\,c^2\,x^8\,\left(3\,b^2-16\right)}{8}+\frac{5\,b\,x^2\,{\left(b^2-16\right)}^4}{512\,c^4}+\frac{5\,x^3\,{\left(b^2-16\right)}^3\,\left(9\,b^2-16\right)}{768\,c^3}+\frac{5\,b\,x^4\,{\left(b^2-16\right)}^2\,\left(3\,b^2-16\right)}{64\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + (b^2/4 - 4)/c)^5,x)
```

```
[Out] (c^5*x^11)/11 + (x*(b^2 - 16)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 1120*b^2 + 3
840))/48 + (5*c*x^7*(21*b^4 - 224*b^2 + 256))/56 + (b*c^4*x^10)/2 + (5*c^3*
x^9*(9*b^2 - 16))/36 + (x^5*(3840*b^2 - 560*b^4 + 21*b^6 - 4096))/(32*c) +
(5*b*c^2*x^8*(3*b^2 - 16))/8 + (5*b*x^2*(b^2 - 16)^4)/(512*c^4) + (5*x^3*(b
^2 - 16)^3*(9*b^2 - 16))/(768*c^3) + (5*b*x^4*(b^2 - 16)^2*(3*b^2 - 16))/(6
4*c^2)
```

3.78 $\int \frac{1}{2+4x+3x^2}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 210}

$$\frac{\text{ArcTan}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x + 3*x^2)^(-1),x]
```

```
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+4x+3x^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 4+6x\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x + 3*x^2)^(-1),x]
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.82, size = 17, normalized size = 0.94

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)}{2}$	17
risch	$\frac{\arctan\left(\frac{(2+3x)\sqrt{2}}{2}\right)\sqrt{2}}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*arctan(1/4*(6*x+4)*2^(1/2))
```

Maxima [A]
time = 0.51, size = 16, normalized size = 0.89

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Fricas [A]
time = 1.34, size = 16, normalized size = 0.89

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Sympy [A]
time = 0.05, size = 22, normalized size = 1.22

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{3\sqrt{2}\,x}{2}+\sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2),x)
```

```
[Out] sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/2
```

Giac [A]
time = 1.59, size = 16, normalized size = 0.89

$$\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\,(3\,x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Mupad [B]
time = 0.14, size = 16, normalized size = 0.89

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,(3\,x+2)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 3*x^2 + 2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(3*x + 2))/2))/2
```

3.79 $\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx$

Optimal. Leaf size=12

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

```
[Out] arctan(x-3^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {632, 210}

$$-\text{ArcTan}\left(\sqrt{3}-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -2\sqrt{3}+2x\right)\right)\\ &= -\tan^{-1}\left(\sqrt{3}-x\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - x]
```

Maple [A]
time = 1.05, size = 9, normalized size = 0.75

method	result	size
default	$\arctan\left(x-\sqrt{3}\right)$	9
risch	$\arctan\left(x-\sqrt{3}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x^2-2*3^(1/2)*x),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x-3^(1/2))
```

Maxima [A]
time = 0.49, size = 8, normalized size = 0.67

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(x - sqrt(3))
```

Fricas [A]
time = 1.33, size = 10, normalized size = 0.83

$$-\arctan\left(-x+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="fricas")
```

```
[Out] -arctan(-x + sqrt(3))
```

Sympy [A]
time = 0.07, size = 7, normalized size = 0.58

$$\operatorname{atan}\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x**2-2*x*3**(1/2)),x)
```

```
[Out] atan(x - sqrt(3))
```

Giac [A]
time = 1.02, size = 8, normalized size = 0.67

$$\arctan\left(x - \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(x - sqrt(3))
```

Mupad [B]
time = 0.27, size = 8, normalized size = 0.67

$$\operatorname{atan}\left(x - \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*3^(1/2)*x + 4),x)
```

```
[Out] atan(x - 3^(1/2))
```

3.80 $\int \frac{1}{2+4x-3x^2} \, dx$

Optimal. Leaf size=19

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

```
[Out] -1/10*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 212}

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x - 3*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(2 - 3*x)/Sqrt[10]]/Sqrt[10])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2+4x-3x^2} \, dx &= -\left(2\text{Subst}\left(\int \frac{1}{40-x^2} \, dx, x, 4-6x\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.79

$$\frac{-\log \left(2+\sqrt{10}-3x\right)+\log \left(-2+\sqrt{10}+3x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x - 3*x^2)^(-1),x]
[Out] (-Log[2 + Sqrt[10] - 3*x] + Log[-2 + Sqrt[10] + 3*x])/(2*Sqrt[10])
```

Maple [A]
time = 0.55, size = 17, normalized size = 0.89

method	result	size
default	$\frac{\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{10}$	17
risch	$\frac{\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{20}-\frac{\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{20}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2),x,method=_RETURNVERBOSE)
[Out] 1/10*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Maxima [A]
time = 0.51, size = 27, normalized size = 1.42

$$-\frac{1}{20}\sqrt{10}\,\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="maxima")
[Out] -1/20*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.
time = 1.98, size = 39, normalized size = 2.05

$$\frac{1}{20}\sqrt{10}\,\log\left(\frac{9\,x^2+2\,\sqrt{10}\,(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(10)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x -
 2))
```

Sympy [A]
time = 0.04, size = 39, normalized size = 2.05

$$\frac{\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{20}-\frac{\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2),x)
```

```
[Out] sqrt(10)*log(x - 2/3 + sqrt(10)/3)/20 - sqrt(10)*log(x - sqrt(10)/3 - 2/3)/
20
```

Giac [A]
time = 0.84, size = 31, normalized size = 1.63

$$-\frac{1}{20}\sqrt{10}\log\left(\frac{\left|6x-2\sqrt{10}-4\right|}{\left|6x+2\sqrt{10}-4\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4))
```

Mupad [B]
time = 0.21, size = 15, normalized size = 0.79

$$\frac{\sqrt{10}\operatorname{atanh}\left(\sqrt{10}\left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 + 2),x)
```

```
[Out] (10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/10
```

3.81 $\int \frac{1}{2+5x+3x^2}\,dx$

Optimal. Leaf size=13

$$-\log(1+x)+\log(2+3x)$$

```
[Out] -ln(1+x)+ln(2+3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{630, 31\}$

$$\log(3x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2)^(-1),x]
```

```
[Out] -Log[1 + x] + Log[2 + 3*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+5x+3x^2}\,dx &= 3\int \frac{1}{2+3x}\,dx-3\int \frac{1}{3+3x}\,dx\\ &= -\log(1+x)+\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\log(1+x)+\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2)^(-1),x]
[Out] -Log[1 + x] + Log[2 + 3*x]
```

Maple [A]
time = 0.45, size = 14, normalized size = 1.08

method	result	size
default	$-\ln\left(x+1\right)+\ln\left(2+3x\right)$	14
norman	$-\ln\left(x+1\right)+\ln\left(2+3x\right)$	14
risch	$-\ln\left(x+1\right)+\ln\left(2+3x\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
[Out] -ln(x+1)+ln(2+3*x)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 1.00

$$\log\left(3\,x+2\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="maxima")
[Out] log(3*x + 2) - log(x + 1)
```

Fricas [A]
time = 1.31, size = 13, normalized size = 1.00

$$\log\left(3\,x+2\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="fricas")
[Out] log(3*x + 2) - log(x + 1)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$\log\left(x+\frac{2}{3}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2),x)
```

```
[Out] log(x + 2/3) - log(x + 1)
```

Giac [A]
time = 0.63, size = 15, normalized size = 1.15

$$\log(|3x + 2|) - \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2),x, algorithm="giac")
```

```
[Out] log(abs(3*x + 2)) - log(abs(x + 1))
```

Mupad [B]
time = 0.08, size = 8, normalized size = 0.62

$$-2 \operatorname{atanh}(6x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 + 2),x)
```

```
[Out] -2*atanh(6*x + 5)
```

3.82 $\int \frac{1}{2+5x-3x^2}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{7}\log(2-x)+\frac{1}{7}\log(1+3x)$$

```
[Out] -1/7*ln(2-x)+1/7*ln(1+3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {630, 31}

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x - 3*x^2)^(-1),x]
```

```
[Out] -1/7*Log[2 - x] + Log[1 + 3*x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+5x-3x^2}\,dx &= -\left(\frac{3}{7}\int \frac{1}{-1-3x}\,dx\right)+\frac{3}{7}\int \frac{1}{6-3x}\,dx\\ &= -\frac{1}{7}\log(2-x)+\frac{1}{7}\log(1+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{1}{7}\log(2-x)+\frac{1}{7}\log(1+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x - 3*x^2)^(-1),x]
```

```
[Out] -1/7*Log[2 - x] + Log[1 + 3*x]/7
```

Maple [A]
time = 0.47, size = 16, normalized size = 0.76

method	result	size
default	$-\frac{\ln(x-2)}{7}+\frac{\ln(3x+1)}{7}$	16
norman	$-\frac{\ln(x-2)}{7}+\frac{\ln(3x+1)}{7}$	16
risch	$-\frac{\ln(x-2)}{7}+\frac{\ln(3x+1)}{7}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*ln(x-2)+1/7*ln(3*x+1)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.71

$$\frac{1}{7}\log\left(3\,x+1\right)-\frac{1}{7}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="maxima")
```

```
[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)
```

Fricas [A]
time = 1.70, size = 15, normalized size = 0.71

$$\frac{1}{7}\log\left(3\,x+1\right)-\frac{1}{7}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="fricas")
```

```
[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.67

$$-\frac{\log\left(x-2\right)}{7}+\frac{\log\left(x+\frac{1}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2),x)
```

```
[Out] -log(x - 2)/7 + log(x + 1/3)/7
```

Giac [A]
time = 0.72, size = 17, normalized size = 0.81

$$\frac{1}{7}\log\left(|3\,x+1|\right)-\frac{1}{7}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2),x, algorithm="giac")
```

```
[Out] 1/7*log(abs(3*x + 1)) - 1/7*log(abs(x - 2))
```

Mupad [B]
time = 0.09, size = 8, normalized size = 0.38

$$\frac{2\,\mathrm{atanh}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x - 3*x^2 + 2),x)
```

```
[Out] (2*atanh((6*x)/7 - 5/7))/7
```

3.83 $\int \frac{1}{3+4x+x^2} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(2+x)$$

```
[Out] -arctanh(2+x)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(6)=12$.
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 2.83, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {630, 31}

$$\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + x^2)^(-1),x]
```

```
[Out] Log[1 + x]/2 - Log[3 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+4x+x^2} \, dx &= \frac{1}{2}\int \frac{1}{1+x} \, dx-\frac{1}{2}\int \frac{1}{3+x} \, dx\\ &= \frac{1}{2}\log(1+x)-\frac{1}{2}\log(3+x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(6)=12$.
time = 0.00, size = 17, normalized size = 2.83

$$\frac{1}{2}\log(1+x)-\frac{1}{2}\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + x^2)^(-1),x]
```

```
[Out] Log[1 + x]/2 - Log[3 + x]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.
time = 0.56, size = 14, normalized size = 2.33

method	result	size
default	$\frac{\ln(x+1)}{2} - \frac{\ln(3+x)}{2}$	14
norman	$\frac{\ln(x+1)}{2} - \frac{\ln(3+x)}{2}$	14
risch	$\frac{\ln(x+1)}{2} - \frac{\ln(3+x)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x+1)-1/2*ln(3+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.
time = 0.29, size = 13, normalized size = 2.17

$$-\frac{1}{2}\log(x+3) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 3) + 1/2*log(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.
time = 1.39, size = 13, normalized size = 2.17

$$-\frac{1}{2}\log(x+3) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="fricas")
```

```
[Out] -1/2*log(x + 3) + 1/2*log(x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.
time = 0.03, size = 12, normalized size = 2.00

$$\frac{\log(x+1)}{2} - \frac{\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4*x+3),x)
```

```
[Out] log(x + 1)/2 - log(x + 3)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.
time = 0.64, size = 15, normalized size = 2.50

$$-\frac{1}{2}\log\left(|x+3|\right)+\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+3),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 3)) + 1/2*log(abs(x + 1))
```

Mupad [B]
time = 0.18, size = 6, normalized size = 1.00

$$-\mathrm{atanh}(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + x^2 + 3),x)
```

```
[Out] -atanh(x + 2)
```

3.84 $\int \frac{1}{1+\pi x+2x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

```
[Out] -2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x + 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x+2x^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-8+\pi^2-x^2}\,dx, x, \pi+4x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x + 2*x^2)^(-1),x]
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Maple [A]
time = 0.52, size = 24, normalized size = 0.89

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\pi+4x}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$	24
risch	$\frac{\ln\left(-\pi^2+\pi\sqrt{\pi^2-8}+4x\sqrt{\pi^2-8}+8\right)}{\sqrt{\pi^2-8}}-\frac{\ln\left(\pi^2+\pi\sqrt{\pi^2-8}+4x\sqrt{\pi^2-8}-8\right)}{\sqrt{\pi^2-8}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

Maxima [A]
time = 0.28, size = 38, normalized size = 1.41

$$\frac{\log\left(\frac{\pi+4\,x-\sqrt{\pi^2-8}}{\pi+4\,x+\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="maxima")
[Out] log((pi + 4*x - sqrt(pi^2 - 8))/(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^2 - 8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.
time = 1.92, size = 50, normalized size = 1.85

$$\frac{\log\left(\frac{\pi^2+4\,\pi x+8\,x^2-(\pi+4\,x)\sqrt{\pi^2-8}-4}{\pi x+2\,x^2+1}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="fricas")
```

```
[Out] log((pi^2 + 4*pi*x + 8*x^2 - (pi + 4*x)*sqrt(pi^2 - 8) - 4)/(pi*x + 2*x^2 +
 1))/sqrt(pi^2 - 8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.
time = 0.10, size = 76, normalized size = 2.81

$$\frac{\log\left(x - \frac{\pi^2}{4\sqrt{-8+\pi^2}} + \frac{\pi}{4} + \frac{2}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}} - \frac{\log\left(x - \frac{2}{\sqrt{-8+\pi^2}} + \frac{\pi}{4} + \frac{\pi^2}{4\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x**2+1),x)
```

```
[Out] log(x - pi**2/(4*sqrt(-8 + pi**2)) + pi/4 + 2/sqrt(-8 + pi**2))/sqrt(-8 + p
i**2) - log(x - 2/sqrt(-8 + pi**2) + pi/4 + pi**2/(4*sqrt(-8 + pi**2)))/sqr
t(-8 + pi**2)
```

Giac [A]
time = 0.71, size = 40, normalized size = 1.48

$$\frac{\log\left(\frac{\left|\pi+4x-\sqrt{\pi^2-8}\right|}{\left|\pi+4x+\sqrt{\pi^2-8}\right|}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="giac")
```

```
[Out] log(abs(pi + 4*x - sqrt(pi^2 - 8))/abs(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^
2 - 8)
```

Mupad [B]
time = 0.36, size = 23, normalized size = 0.85

$$-\frac{2\,\mathrm{atanh}\left(\frac{\Pi+4\,x}{\sqrt{\Pi^2-8}}\right)}{\sqrt{\Pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x + 2*x^2 + 1),x)
```

```
[Out] -(2*atanh((Pi + 4*x)/(Pi^2 - 8)^(1/2)))/(Pi^2 - 8)^(1/2)
```

3.85 $\int \frac{1}{1+\pi x-2x^2} dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

```
[Out] -2*arctanh((Pi-4*x)/(Pi^2+8)^(1/2))/(Pi^2+8)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x - 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi - 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x-2x^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{8+\pi^2-x^2}\,dx, x, \pi-4x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.07

$$\frac{2\tanh^{-1}\left(\frac{-\pi+4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x - 2*x^2)^(-1),x]
[Out] (2*ArcTanh[(-Pi + 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Maple [A]
time = 0.58, size = 26, normalized size = 0.96

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{-\pi+4x}{\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$	26
risch	$\frac{\ln\left(\pi^2-\pi\sqrt{\pi^2+8}+4x\sqrt{\pi^2+8}+8\right)}{\sqrt{\pi^2+8}}-\frac{\ln\left(-\pi^2-\pi\sqrt{\pi^2+8}+4x\sqrt{\pi^2+8}-8\right)}{\sqrt{\pi^2+8}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/(Pi^2+8)^(1/2)*arctanh((-Pi+4*x)/(Pi^2+8)^(1/2))
```

Maxima [A]
time = 0.27, size = 39, normalized size = 1.44

$$-\frac{\log\left(\frac{\pi-4x+\sqrt{\pi^2+8}}{\pi-4x-\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="maxima")
[Out] -log((pi - 4*x + sqrt(pi^2 + 8))/(pi - 4*x - sqrt(pi^2 + 8)))/sqrt(pi^2 + 8
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.
time = 0.97, size = 51, normalized size = 1.89

$$\frac{\log\left(-\frac{\pi^2-4\pi x+8x^2-(\pi-4x)\sqrt{\pi^2+8}+4}{\pi x-2x^2+1}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="fricas")
```

```
[Out] log(-(pi^2 - 4*pi*x + 8*x^2 - (pi - 4*x)*sqrt(pi^2 + 8) + 4)/(pi*x - 2*x^2
+ 1))/sqrt(pi^2 + 8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.
time = 0.12, size = 76, normalized size = 2.81

$$-\frac{\log\left(x-\frac{\pi}{4}-\frac{\pi^2}{4\sqrt{8+\pi^2}}-\frac{2}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}+\frac{\log\left(x-\frac{\pi}{4}+\frac{2}{\sqrt{8+\pi^2}}+\frac{\pi^2}{4\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x**2+1),x)
```

```
[Out] -log(x - pi/4 - pi**2/(4*sqrt(8 + pi**2)) - 2/sqrt(8 + pi**2))/sqrt(8 + pi*
*2) + log(x - pi/4 + 2/sqrt(8 + pi**2) + pi**2/(4*sqrt(8 + pi**2)))/sqrt(8
+ pi**2)
```

Giac [A]
time = 0.68, size = 45, normalized size = 1.67

$$-\frac{\log\left(\frac{\left|-\pi+4x-\sqrt{\pi^2+8}\right|}{\left|-\pi+4x+\sqrt{\pi^2+8}\right|}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-2*x^2+1),x, algorithm="giac")
```

```
[Out] -log(abs(-pi + 4*x - sqrt(pi^2 + 8))/abs(-pi + 4*x + sqrt(pi^2 + 8)))/sqrt(
pi^2 + 8)
```

Mupad [B]
time = 0.39, size = 23, normalized size = 0.85

$$-\frac{2\,\mathrm{atanh}\left(\frac{\Pi-4\,x}{\sqrt{\Pi^2+8}}\right)}{\sqrt{\Pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x - 2*x^2 + 1),x)
```

```
[Out] -(2*atanh((Pi - 4*x)/(Pi^2 + 8)^(1/2)))/(Pi^2 + 8)^(1/2)
```

3.86 $\int \frac{1}{1+\pi x+3x^2} \, dx$

Optimal. Leaf size=31

$$\frac{2\tan^{-1}\left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

```
[Out] 2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 210}

$$\frac{2\text{ArcTan}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x + 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x+3x^2} \, dx &= -\left(2\text{Subst}\left(\int \frac{1}{-12+\pi^2-x^2} \, dx, x, \pi+6x\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\pi+6x}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x + 3*x^2)^(-1),x]
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Maple [A]
time = 1.00, size = 28, normalized size = 0.90

method	result	size
default	$\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$	28
risch	$\frac{\ln\left(-\pi^2+\pi\sqrt{\pi^2-12}+6x\sqrt{\pi^2-12}+12\right)}{\sqrt{\pi^2-12}}-\frac{\ln\left(\pi^2+\pi\sqrt{\pi^2-12}+6x\sqrt{\pi^2-12}-12\right)}{\sqrt{\pi^2-12}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x+3*x^2+1),x,method=_RETURNVERBOSE)
[Out] 2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

Maxima [A]
time = 0.32, size = 27, normalized size = 0.87

$$\frac{2\ \arctan\left(\frac{\pi+6\,x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="maxima")
[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

Fricas [A]
time = 2.79, size = 41, normalized size = 1.32

$$\frac{2\sqrt{-\pi^2+12}\ \arctan\left(\frac{(\pi+6\,x)\sqrt{-\pi^2+12}}{\pi^2-12}\right)}{\pi^2-12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-pi^2 + 12)*arctan((pi + 6*x)*sqrt(-pi^2 + 12)/(pi^2 - 12))/(pi^2 -
12)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.08, size = 87, normalized size = 2.81

$$-\frac{i\log\left(x+\frac{\pi}{6}-\frac{2i}{\sqrt{12-\pi^2}}+\frac{i\pi^2}{6\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}+\frac{i\log\left(x+\frac{\pi}{6}-\frac{i\pi^2}{6\sqrt{12-\pi^2}}+\frac{2i}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x**2+1),x)
```

```
[Out] -I*log(x + pi/6 - 2*I/sqrt(12 - pi**2) + I*pi**2/(6*sqrt(12 - pi**2)))/sqrt
(12 - pi**2) + I*log(x + pi/6 - I*pi**2/(6*sqrt(12 - pi**2)) + 2*I/sqrt(12
- pi**2))/sqrt(12 - pi**2)
```

Giac [A]
time = 0.82, size = 27, normalized size = 0.87

$$\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="giac")
```

```
[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

Mupad [B]
time = 0.38, size = 23, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\Pi+6\,x}{\sqrt{\Pi^2-12}}\right)}{\sqrt{\Pi^2-12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x + 3*x^2 + 1),x)
```

```
[Out] -(2*atanh((Pi + 6*x)/(Pi^2 - 12)^(1/2)))/(Pi^2 - 12)^(1/2)
```

3.87 $\int \frac{1}{1+\pi x-3x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

```
[Out] -2*arctanh((Pi-6*x)/(Pi^2+12)^(1/2))/(Pi^2+12)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Pi*x - 3*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi - 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\pi x-3x^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{12+\pi^2-x^2}\,dx,x,\pi-6x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.07

$$\frac{2\tanh^{-1}\left(\frac{-\pi+6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Pi*x - 3*x^2)^(-1),x]
[Out] (2*ArcTanh[(-Pi + 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Maple [A]
time = 0.59, size = 26, normalized size = 0.96

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{-\pi+6x}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$	26
risch	$\frac{\ln\left(\pi^2-\pi\sqrt{\pi^2+12}+6x\sqrt{\pi^2+12}+12\right)}{\sqrt{\pi^2+12}}-\frac{\ln\left(-\pi^2-\pi\sqrt{\pi^2+12}+6x\sqrt{\pi^2+12}-12\right)}{\sqrt{\pi^2+12}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x-3*x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/(Pi^2+12)^(1/2)*arctanh((-Pi+6*x)/(Pi^2+12)^(1/2))
```

Maxima [A]
time = 0.29, size = 39, normalized size = 1.44

$$-\frac{\log\left(\frac{\pi-6\,x+\sqrt{\pi^2+12}}{\pi-6\,x-\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="maxima")
[Out] -log((pi - 6*x + sqrt(pi^2 + 12))/(pi - 6*x - sqrt(pi^2 + 12)))/sqrt(pi^2 +
 12)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.
time = 1.94, size = 51, normalized size = 1.89

$$\frac{\log\left(-\frac{\pi^2-6\,\pi x+18\,x^2-(\pi-6\,x)\sqrt{\pi^2+12}+6}{\pi x-3\,x^2+1}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="fricas")
```

```
[Out] log(-(pi^2 - 6*pi*x + 18*x^2 - (pi - 6*x)*sqrt(pi^2 + 12) + 6)/(pi*x - 3*x^
2 + 1))/sqrt(pi^2 + 12)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.
time = 0.13, size = 76, normalized size = 2.81

$$\frac{\log\left(x - \frac{\pi}{6} + \frac{\pi^2}{6\sqrt{\pi^2+12}} + \frac{2}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}} - \frac{\log\left(x - \frac{\pi}{6} - \frac{2}{\sqrt{\pi^2+12}} - \frac{\pi^2}{6\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x**2+1),x)
```

```
[Out] log(x - pi/6 + pi**2/(6*sqrt(pi**2 + 12)) + 2/sqrt(pi**2 + 12))/sqrt(pi**2
+ 12) - log(x - pi/6 - 2/sqrt(pi**2 + 12) - pi**2/(6*sqrt(pi**2 + 12)))/sqr
t(pi**2 + 12)
```

Giac [A]
time = 0.71, size = 45, normalized size = 1.67

$$-\frac{\log\left(\frac{\left|-\pi+6\,x-\sqrt{\pi^2+12}\right|}{\left|-\pi+6\,x+\sqrt{\pi^2+12}\right|}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(pi*x-3*x^2+1),x, algorithm="giac")
```

```
[Out] -log(abs(-pi + 6*x - sqrt(pi^2 + 12))/abs(-pi + 6*x + sqrt(pi^2 + 12)))/sqr
t(pi^2 + 12)
```

Mupad [B]
time = 0.42, size = 23, normalized size = 0.85

$$-\frac{2\,\mathrm{atanh}\left(\frac{\Pi-6\,x}{\sqrt{\Pi^2+12}}\right)}{\sqrt{\Pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi*x - 3*x^2 + 1),x)
```

```
[Out] -(2*atanh((Pi - 6*x)/(Pi^2 + 12)^(1/2)))/(Pi^2 + 12)^(1/2)
```

3.88 $\int \frac{1}{a+cx+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

```
[Out] 2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 210}

$$\frac{2\text{ArcTan}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x + b*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+cx+bx^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-4ab+c^2-x^2}\,dx, x, c+2bx\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x + b*x^2)^(-1),x]
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Maple [A]
time = 0.64, size = 35, normalized size = 0.92

method	result	size
default	$\frac{2\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$	35
risch	$-\frac{\ln\left(2bx+\sqrt{-4ab+c^2}+c\right)}{\sqrt{-4ab+c^2}}+\frac{\ln\left(-2bx+\sqrt{-4ab+c^2}-c\right)}{\sqrt{-4ab+c^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+c*x+a),x,method=_RETURNVERBOSE)
[Out] 2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 1.40, size = 113, normalized size = 2.97

$$\left[-\frac{\sqrt{-4\,ab+c^2}\,\log\left(\frac{2\,b^2x^2+2\,bcx-2\,ab+c^2-\sqrt{-4\,ab+c^2}\,(2\,bx+c)}{bx^2+cx+a}\right)}{4\,ab-c^2},\,-\frac{2\,\arctan\left(-\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="fricas")
```

```
[Out] [-sqrt(-4*a*b + c^2)*log((2*b^2*x^2 + 2*b*c*x - 2*a*b + c^2 - sqrt(-4*a*b +
 c^2)*(2*b*x + c))/(b*x^2 + c*x + a))/(4*a*b - c^2), -2*arctan(-(2*b*x + c)
/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(32) = 64$.
time = 0.09, size = 124, normalized size = 3.26

$$-\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{-4ab\sqrt{-\frac{1}{4ab-c^2}}+c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)+\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{4ab\sqrt{-\frac{1}{4ab-c^2}}-c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+c*x+a),x)
```

```
[Out] -sqrt(-1/(4*a*b - c**2))*log(x + (-4*a*b*sqrt(-1/(4*a*b - c**2)) + c**2*sqr
t(-1/(4*a*b - c**2)) + c)/(2*b)) + sqrt(-1/(4*a*b - c**2))*log(x + (4*a*b*s
qrt(-1/(4*a*b - c**2)) - c**2*sqrt(-1/(4*a*b - c**2)) + c)/(2*b))
```

Giac [A]
time = 1.30, size = 34, normalized size = 0.89

$$\frac{2\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{\sqrt{4\,ab-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a),x, algorithm="giac")
```

```
[Out] 2*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)
```

Mupad [B]
time = 0.23, size = 46, normalized size = 1.21

$$\frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{4\,a\,b-c^2}}+\frac{2\,b\,x}{\sqrt{4\,a\,b-c^2}}\right)}{\sqrt{4\,a\,b-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x + b*x^2),x)
```

```
[Out] (2*atan(c/(4*a*b - c^2)^(1/2) + (2*b*x)/(4*a*b - c^2)^(1/2)))/(4*a*b - c^2)
^(1/2)
```

3.89 $\int \frac{1}{b+2ax+bx^2} \, dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] -arctanh((b*x+a)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {632, 212}

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x + b*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(a + b*x)/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b+2ax+bx^2} \, dx &= -\left(2\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2\right)-x^2} \, dx, x, 2a+2bx\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.97

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x + b*x^2)^(-1),x]
[Out] ArcTan[(a + b*x)/Sqrt[-a^2 + b^2]]/Sqrt[-a^2 + b^2]
```

Maple [A]
time = 0.64, size = 35, normalized size = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	35
risch	$\frac{\ln\left(-bx+\sqrt{a^2-b^2}-a\right)}{2\sqrt{a^2-b^2}}-\frac{\ln\left(bx+\sqrt{a^2-b^2}+a\right)}{2\sqrt{a^2-b^2}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2*a*x+b),x,method=_RETURNVERBOSE)
[Out] 1/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 1.35, size = 124, normalized size = 3.54

$$\left[\frac{\log\left(\frac{b^2x^2+2\,abx+2\,a^2-b^2-2\,\sqrt{a^2-b^2}\,(bx+a)}{bx^2+2\,ax+b}\right)}{2\,\sqrt{a^2-b^2}},-\frac{\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(bx+a)}{a^2-b^2}\right)}{a^2-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b
*x^2 + 2*a*x + b))/sqrt(a^2 - b^2), -sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*x + a)/(a^2 - b^2))/(a^2 - b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(27) = 54$.
time = 0.11, size = 100, normalized size = 2.86

$$\frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a+b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}-\frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a-b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2*a*x+b),x)
```

```
[Out] sqrt(1/((a - b)*(a + b)))*log(x + (-a**2*sqrt(1/((a - b)*(a + b))) + a + b*
*2*sqrt(1/((a - b)*(a + b))))/b)/2 - sqrt(1/((a - b)*(a + b)))*log(x + (a**
2*sqrt(1/((a - b)*(a + b))) + a - b**2*sqrt(1/((a - b)*(a + b))))/b)/2
```

Giac [A]
time = 1.46, size = 30, normalized size = 0.86

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="giac")
```

```
[Out] arctan((b*x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

Mupad [B]
time = 0.27, size = 33, normalized size = 0.94

$$-\frac{\operatorname{atanh}\left(\frac{a+b\,x}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x + b*x^2),x)
```

```
[Out] -atanh((a + b*x)/((a + b)^(1/2)*(a - b)^(1/2)))/((a + b)^(1/2)*(a - b)^(1/2
))
```

3.90 $\int \frac{1}{b+2ax-bx^2}\,dx$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -arctanh((-b*x+a)/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {632, 212}

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x - b*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(a - b*x)/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b+2ax-bx^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2a-2bx\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.28

$$-\frac{\tan^{-1}\left(\frac{-a+bx}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x - b*x^2)^(-1),x]
[Out] -(ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]]/Sqrt[-a^2 - b^2])
```

Maple [A]
time = 0.60, size = 32, normalized size = 1.00

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{-2bx+2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	32
risch	$\frac{\ln\left(bx+\sqrt{a^2+b^2}-a\right)}{2\sqrt{a^2+b^2}}-\frac{\ln\left(-bx+\sqrt{a^2+b^2}+a\right)}{2\sqrt{a^2+b^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2*a*x+b),x,method=_RETURNVERBOSE)
[Out] -1/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*x+2*a)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.52, size = 49, normalized size = 1.53

$$-\frac{\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="maxima")
[Out] -1/2*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/sqrt(a^2
+ b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.
time = 1.71, size = 67, normalized size = 2.09

$$\frac{\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}\,(bx-a)}{bx^2-2\,ax-b}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="fricas")
```

```
[Out] 1/2*log((b^2*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*
x^2 - 2*a*x - b))/sqrt(a^2 + b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(27) = 54$.

time = 0.11, size = 102, normalized size = 3.19

$$-\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{a^2+b^2}}-a-b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}+\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{a^2\sqrt{\frac{1}{a^2+b^2}}-a+b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2*a*x+b),x)
```

```
[Out] -sqrt(1/(a**2 + b**2))*log(x + (-a**2*sqrt(1/(a**2 + b**2)) - a - b**2*sqrt
(1/(a**2 + b**2)))/b)/2 + sqrt(1/(a**2 + b**2))*log(x + (a**2*sqrt(1/(a**2
+ b**2)) - a + b**2*sqrt(1/(a**2 + b**2)))/b)/2
```

Giac [A]

time = 1.42, size = 55, normalized size = 1.72

$$-\frac{\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^2
+ b^2)))/sqrt(a^2 + b^2)
```

Mupad [B]

time = 0.23, size = 28, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\frac{a-b\,x}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x - b*x^2),x)
```

```
[Out] -atanh((a - b*x)/(a^2 + b^2)^(1/2))/(a^2 + b^2)^(1/2)
```

3.91 $\int \frac{1}{(2+4x+3x^2)^2}\,dx$

Optimal. Leaf size=43

$$\frac{2+3x}{4\left(2+4x+3x^2\right)}+\frac{3\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

[Out] 1/4*(2+3*x)/(3*x^2+4*x+2)+3/8*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {628, 632, 210}

$$\frac{3\text{ArcTan}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{3x+2}{4\left(3x^2+4x+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x + 3*x^2)^(-2),x]
```

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(2+4x+3x^2)^2}\,dx = \frac{2+3x}{4(2+4x+3x^2)} + \frac{3}{4}\int \frac{1}{2+4x+3x^2}\,dx$$
$$= \frac{2+3x}{4(2+4x+3x^2)} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 4+6x\right)$$
$$= \frac{2+3x}{4(2+4x+3x^2)} + \frac{3\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.00

$$\frac{2+3x}{4(2+4x+3x^2)} + \frac{3\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x + 3*x^2)^(-2),x]
```

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])
```

Maple [A]
time = 0.64, size = 37, normalized size = 0.86

method	result	size
risch	$\frac{\frac{x}{4}+\frac{1}{6}}{x^2+\frac{4}{3}x+\frac{2}{3}} + \frac{3\arctan\left(\frac{(2+3x)\sqrt{2}}{2}\right)\sqrt{2}}{8}$	34
default	$\frac{6x+4}{24x^2+32x+16} + \frac{3\sqrt{2}\,\arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)}{8}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(3*x^2+4*x+2)*(6*x+4)+3/8*2^(1/2)*arctan(1/4*(6*x+4)*2^(1/2))
```

Maxima [A]
time = 0.51, size = 36, normalized size = 0.84

$$\frac{3}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right) + \frac{3\,x+2}{4\,(3\,x^2+4\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2)
```

Fricas [A]
time = 1.19, size = 45, normalized size = 1.05

$$\frac{3\sqrt{2}\,(3\,x^2+4\,x+2)\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)+6\,x+4}{8\,(3\,x^2+4\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(3*x^2 + 4*x + 2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 6*x + 4)/(
3*x^2 + 4*x + 2)
```

Sympy [A]
time = 0.05, size = 39, normalized size = 0.91

$$\frac{3x+2}{12x^2+16x+8}+\frac{3\sqrt{2}\,\operatorname{atan}\left(\frac{3\sqrt{2}\,x}{2}+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2)**2,x)
```

```
[Out] (3*x + 2)/(12*x**2 + 16*x + 8) + 3*sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/8
```

Giac [A]
time = 0.86, size = 36, normalized size = 0.84

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(3\,x+2)\right)+\frac{3\,x+2}{4\,(3\,x^2+4\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2)
```

Mupad [B]
time = 0.04, size = 33, normalized size = 0.77

$$\frac{\frac{x}{4}+\frac{1}{6}}{x^2+\frac{4\,x}{3}+\frac{2}{3}}+\frac{3\sqrt{2}\,\operatorname{atan}\left(\frac{3\sqrt{2}\,x}{2}+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 3*x^2 + 2)^2,x)
```

```
[Out] (x/4 + 1/6)/((4*x)/3 + x^2 + 2/3) + (3*2^(1/2)*atan((3*2^(1/2)*x)/2 + 2^(1/
2)))/8
```

3.92 $\int \frac{1}{(2+4x-3x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{2-3x}{20\left(2+4x-3x^2\right)}-\frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

```
[Out] 1/20*(-2+3*x)/(-3*x^2+4*x+2)-3/200*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {628, 632, 212}

$$-\frac{2-3x}{20\left(-3x^2+4x+2\right)}-\frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] -1/20*(2 - 3*x)/(2 + 4*x - 3*x^2) - (3*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sqr
t[10])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(2+4x-3x^2)^2}\,dx = -\frac{2-3x}{20\,(2+4x-3x^2)} + \frac{3}{20}\int \frac{1}{2+4x-3x^2}\,dx$$
$$= -\frac{2-3x}{20\,(2+4x-3x^2)} - \frac{3}{10}\text{Subst}\left(\int \frac{1}{40-x^2}\,dx, x, 4-6x\right)$$
$$= -\frac{2-3x}{20\,(2+4x-3x^2)} - \frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 1.44

$$\frac{2-3x}{20\,(-2-4x+3x^2)} - \frac{3\log\left(2+\sqrt{10}-3x\right)}{40\sqrt{10}} + \frac{3\log\left(-2+\sqrt{10}+3x\right)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] (2 - 3*x)/(20*(-2 - 4*x + 3*x^2)) - (3*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[10
]) + (3*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

Maple [A]
time = 0.56, size = 37, normalized size = 0.86

method	result	size
default	$-\frac{6x-4}{40(3x^2-4x-2)}+\frac{3\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200}$	37
risch	$\frac{-\frac{x}{20}+\frac{1}{30}}{x^2-\frac{4}{3}x-\frac{2}{3}}+\frac{3\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{400}-\frac{3\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{400}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(6*x-4)/(3*x^2-4*x-2)+3/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Maxima [A]
time = 0.49, size = 47, normalized size = 1.09

$$-\frac{3}{400}\,\sqrt{10}\,\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right) - \frac{3\,x-2}{20\,(3\,x^2-4\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="maxima")
```

```
[Out] -3/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(3*x
- 2)/(3*x^2 - 4*x - 2)
```

Fricas [A]
time = 1.71, size = 68, normalized size = 1.58

$$\frac{3\sqrt{10}\,(3\,x^2-4\,x-2)\log\left(\frac{9\,x^2+2\,\sqrt{10}\,(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right)-60\,x+40}{400\,(3\,x^2-4\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(3*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*
x + 14)/(3*x^2 - 4*x - 2)) - 60*x + 40)/(3*x^2 - 4*x - 2)
```

Sympy [A]
time = 0.07, size = 58, normalized size = 1.35

$$\frac{2-3x}{60x^2-80x-40}+\frac{3\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400}-\frac{3\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2)**2,x)
```

```
[Out] (2 - 3*x)/(60*x**2 - 80*x - 40) + 3*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/400
- 3*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

Giac [A]
time = 0.64, size = 51, normalized size = 1.19

$$-\frac{3}{400}\,\sqrt{10}\,\log\left(\frac{\left|6\,x-2\,\sqrt{10}-4\right|}{\left|6\,x+2\,\sqrt{10}-4\right|}\right)-\frac{3\,x-2}{20\,(3\,x^2-4\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="giac")
```

```
[Out] -3/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
1/20*(3*x - 2)/(3*x^2 - 4*x - 2)
```

Mupad [B]
time = 0.16, size = 34, normalized size = 0.79

$$\frac{3\sqrt{10}\,\operatorname{atanh}\left(\sqrt{10}\,\left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{200}+\frac{\frac{x}{20}-\frac{1}{30}}{-x^2+\frac{4x}{3}+\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 + 2)^2,x)
[Out] (3*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + (x/20 - 1/30)/((4*x)/3
- x^2 + 2/3)
```

3.93 $\int \frac{1}{(2+5x+3x^2)^2} \, dx$

Optimal. Leaf size=34

$$-\frac{5+6x}{2+5x+3x^2}+6\log(1+x)-6\log(2+3x)$$

```
[Out] (-5-6*x)/(3*x^2+5*x+2)+6*ln(1+x)-6*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {628, 630, 31}

$$-\frac{6x+5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2)^(-2),x]
```

```
[Out] -((5 + 6*x)/(2 + 5*x + 3*x^2)) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(2+5x+3x^2)^2}\,dx = -\frac{5+6x}{2+5x+3x^2} - 6\int \frac{1}{2+5x+3x^2}\,dx$$
$$= -\frac{5+6x}{2+5x+3x^2} - 18\int \frac{1}{2+3x}\,dx + 18\int \frac{1}{3+3x}\,dx$$
$$= -\frac{5+6x}{2+5x+3x^2} + 6\log(1+x) - 6\log(2+3x)$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.97

$$\frac{-5-6x}{2+5x+3x^2} + 6\log(1+x) - 6\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2)^(-2),x]
[Out] (-5 - 6*x)/(2 + 5*x + 3*x^2) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

Maple [A]
time = 0.61, size = 32, normalized size = 0.94

method	result	size
default	$-\frac{1}{x+1} + 6\ln(x+1) - \frac{3}{2+3x} - 6\ln(2+3x)$	32
risch	$\frac{-2x-\frac{5}{3}}{x^2+\frac{5}{3}x+\frac{2}{3}} + 6\ln(x+1) - 6\ln(2+3x)$	32
norman	$\frac{\frac{15}{2}x^2+\frac{13}{2}x}{3x^2+5x+2} + 6\ln(x+1) - 6\ln(2+3x)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2)^2,x,method=_RETURNVERBOSE)
[Out] -1/(x+1)+6*ln(x+1)-3/(2+3*x)-6*ln(2+3*x)
```

Maxima [A]
time = 0.27, size = 34, normalized size = 1.00

$$-\frac{6\,x+5}{3\,x^2+5\,x+2} - 6\,\log(3\,x+2) + 6\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="maxima")
[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(3*x + 2) + 6*log(x + 1)
```

Fricas [A]
time = 1.35, size = 53, normalized size = 1.56

$$-\frac{6\,(3\,x^2+5\,x+2)\log\left(3\,x+2\right)-6\,(3\,x^2+5\,x+2)\log\left(x+1\right)+6\,x+5}{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="fricas")
```

```
[Out] -(6*(3*x^2 + 5*x + 2)*log(3*x + 2) - 6*(3*x^2 + 5*x + 2)*log(x + 1) + 6*x +
 5)/(3*x^2 + 5*x + 2)
```

Sympy [A]
time = 0.05, size = 31, normalized size = 0.91

$$\frac{-6x-5}{3x^2+5x+2}-6\log\left(x+\frac{2}{3}\right)+6\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2)**2,x)
```

```
[Out] (-6*x - 5)/(3*x**2 + 5*x + 2) - 6*log(x + 2/3) + 6*log(x + 1)
```

Giac [A]
time = 0.70, size = 36, normalized size = 1.06

$$-\frac{6\,x+5}{3\,x^2+5\,x+2}-6\,\log\left(|3\,x+2|\right)+6\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="giac")
```

```
[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(abs(3*x + 2)) + 6*log(abs(x + 1))
```

Mupad [B]
time = 0.20, size = 34, normalized size = 1.00

$$-6\,\ln\left(\frac{3\,x+2}{x+1}\right)-\frac{2\,\left(3\,x+\frac{5}{2}\right)}{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 + 2)^2,x)
```

```
[Out] - 6*log((3*x + 2)/(x + 1)) - (2*(3*x + 5/2))/(5*x + 3*x^2 + 2)
```

3.94 $\int \frac{1}{(2+5x-3x^2)^2} \, dx$

Optimal. Leaf size=42

$$-\frac{5-6x}{49\left(2+5x-3x^2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(1+3x)$$

```
[Out] 1/49*(-5+6*x)/(-3*x^2+5*x+2)-6/343*ln(2-x)+6/343*ln(1+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {628, 630, 31}

$$-\frac{5-6x}{49\left(-3x^2+5x+2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x - 3*x^2)^(-2),x]
```

```
[Out] -1/49*(5 - 6*x)/(2 + 5*x - 3*x^2) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/3
43
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(2+5x-3x^2)^2}\,dx = -\frac{5-6x}{49\,(2+5x-3x^2)} + \frac{6}{49}\int \frac{1}{2+5x-3x^2}\,dx$$
$$= -\frac{5-6x}{49\,(2+5x-3x^2)} - \frac{18}{343}\int \frac{1}{-1-3x}\,dx + \frac{18}{343}\int \frac{1}{6-3x}\,dx$$
$$= -\frac{5-6x}{49\,(2+5x-3x^2)} - \frac{6}{343}\log(2-x) + \frac{6}{343}\log(1+3x)$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$\frac{5-6x}{49\,(-2-5x+3x^2)} - \frac{6}{343}\log(2-x) + \frac{6}{343}\log(1+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x - 3*x^2)^(-2),x]
[Out] (5 - 6*x)/(49*(-2 - 5*x + 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/3
43
```

Maple [A]
time = 0.53, size = 32, normalized size = 0.76

method	result	size
default	$-\frac{1}{49(x-2)}-\frac{6\ln(x-2)}{343}-\frac{3}{49(3x+1)}+\frac{6\ln(3x+1)}{343}$	32
risch	$\frac{-\frac{2x}{49}+\frac{5}{147}}{x^2-\frac{5}{3}x-\frac{2}{3}}-\frac{6\ln(x-2)}{343}+\frac{6\ln(3x+1)}{343}$	32
norman	$\frac{\frac{15}{98}x^2-\frac{37}{98}x}{3x^2-5x-2}-\frac{6\ln(x-2)}{343}+\frac{6\ln(3x+1)}{343}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2)^2,x,method=_RETURNVERBOSE)
[Out] -1/49/(x-2)-6/343*ln(x-2)-3/49/(3*x+1)+6/343*ln(3*x+1)
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.81

$$-\frac{6\,x-5}{49\,(3\,x^2-5\,x-2)} + \frac{6}{343}\,\log\left(3\,x+1\right) - \frac{6}{343}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="maxima")
```

```
[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(3*x + 1) - 6/343*log(x - 2)
```

Fricas [A]
time = 2.29, size = 53, normalized size = 1.26

$$\frac{6\,(3\,x^2-5\,x-2)\log\left(3\,x+1\right)-6\,(3\,x^2-5\,x-2)\log\left(x-2\right)-42\,x+35}{343\,(3\,x^2-5\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/343*(6*(3*x^2 - 5*x - 2)*log(3*x + 1) - 6*(3*x^2 - 5*x - 2)*log(x - 2) -
42*x + 35)/(3*x^2 - 5*x - 2)
```

Sympy [A]
time = 0.06, size = 32, normalized size = 0.76

$$\frac{5-6x}{147x^2-245x-98}-\frac{6\log\left(x-2\right)}{343}+\frac{6\log\left(x+\frac{1}{3}\right)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2)**2,x)
```

```
[Out] (5 - 6*x)/(147*x**2 - 245*x - 98) - 6*log(x - 2)/343 + 6*log(x + 1/3)/343
```

Giac [A]
time = 0.66, size = 36, normalized size = 0.86

$$-\frac{6\,x-5}{49\,(3\,x^2-5\,x-2)}+\frac{6}{343}\,\log\left(|3\,x+1|\right)-\frac{6}{343}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="giac")
```

```
[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(abs(3*x + 1)) - 6/343*log(abs
(x - 2))
```

Mupad [B]
time = 0.08, size = 34, normalized size = 0.81

$$\frac{6\,\ln\left(\frac{3\,x+1}{x-2}\right)}{343}+\frac{2\,\left(3\,x-\frac{5}{2}\right)}{49\,\left(-3\,x^2+5\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x - 3*x^2 + 2)^2,x)
```

```
[Out] (6*log((3*x + 1)/(x - 2)))/343 + (2*(3*x - 5/2))/(49*(5*x - 3*x^2 + 2))
```

3.95 $\int \frac{1}{\left(a+cx+bx^2\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{c+2bx}{\left(4ab-c^2\right)\left(a+cx+bx^2\right)}+\frac{4b\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}$$

```
[Out] (2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))
/(4*a*b-c^2)^(3/2)
```

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {628, 632, 210}

$$\frac{4b\text{ArcTan}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\left(4ab-c^2\right)^{3/2}}+\frac{2bx+c}{\left(4ab-c^2\right)\left(a+bx^2+cx\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x + b*x^2)^(-2),x]
```

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*ArcTan[(c + 2*b*x)/Sqr
t[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+cx+bx^2)^2}\,dx &= \frac{c+2bx}{(4ab-c^2)(a+cx+bx^2)}+\frac{(2b)\int \frac{1}{a+cx+bx^2}\,dx}{4ab-c^2}\\ &= \frac{c+2bx}{(4ab-c^2)(a+cx+bx^2)}-\frac{(4b)\text{Subst}\left(\int \frac{1}{-4ab+c^2-x^2}\,dx,x,c+2bx\right)}{4ab-c^2}\\ &= \frac{c+2bx}{(4ab-c^2)(a+cx+bx^2)}+\frac{4b\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 70, normalized size = 0.99

$$\frac{c+2bx}{(4ab-c^2)(a+x(c+bx))}+\frac{4b\tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x + b*x^2)^(-2),x]
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + x*(c + b*x))) + (4*b*ArcTan[(c + 2*b*x)/Sqr
t[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Maple [A]
time = 0.61, size = 68, normalized size = 0.96

method	result	size
default	$\frac{2bx+c}{(4ab-c^2)(b\,x^2+cx+a)}+\frac{4b\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2bx}{4ab-c^2}+\frac{c}{4ab-c^2}}{b\,x^2+cx+a}+\frac{2b\ln\left((-8a\,b^2+2b\,c^2)x+(-4ab+c^2)^{\frac{3}{2}}-4abc+c^3\right)}{(-4ab+c^2)^{\frac{3}{2}}}-\frac{2b\ln\left((8a\,b^2-2b\,c^2)x+(-4ab+c^2)^{\frac{3}{2}}+4abc-c^3\right)}{(-4ab+c^2)^{\frac{3}{2}}}$	144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+c*x+a)^2,x,method=_RETURNVERBOSE)
[Out] (2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))
/(4*a*b-c^2)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(67) = 134$.
time = 2.17, size = 334, normalized size = 4.70

$$\left[\frac{4\,abc - c^{3} + 2\,(b^{2}x^{2} + bcx + ab)\sqrt{-4\,ab + c^{2}}\,\log\left(\frac{2\,b^{2}x^{2} + 2\,bcx - 2\,ab + c^{2} + \sqrt{-4\,ab + c^{2}}\,(2\,bx + c)}{bx^{2} + cx + a}\right) + 2\,(4\,ab^{2} - bc^{2})x}{16\,a^{3}b^{2} - 8\,a^{2}bc^{2} + ac^{4} + (16\,a^{2}b^{3} - 8\,ab^{2}c^{2} + bc^{4})x^{2} + (16\,a^{2}b^{2}c - 8\,abc^{3} + c^{5})x}, \frac{4\,abc - c^{3} - 4\,(b^{2}x^{2} + bcx + ab)\sqrt{4\,ab - c^{2}}\,\arctan\left(-\frac{2\,bx + c}{\sqrt{4\,ab - c^{2}}}\right) + 2\,(4\,ab^{2} - bc^{2})x}{16\,a^{3}b^{2} - 8\,a^{2}bc^{2} + ac^{4} + (16\,a^{2}b^{3} - 8\,ab^{2}c^{2} + bc^{4})x^{2} + (16\,a^{2}b^{2}c - 8\,abc^{3} + c^{5})x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="fricas")
```

```
[Out] [(4*a*b*c - c^3 + 2*(b^2*x^2 + b*c*x + a*b)*sqrt(-4*a*b + c^2)*log((2*b^2*x
^2 + 2*b*c*x - 2*a*b + c^2 + sqrt(-4*a*b + c^2)*(2*b*x + c))/(b*x^2 + c*x +
 a)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c^4 + (16*a^2*b
^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*c^3 + c^5)*x), (4*a*b
*c - c^3 - 4*(b^2*x^2 + b*c*x + a*b)*sqrt(4*a*b - c^2)*arctan(-(2*b*x + c)/
sqrt(4*a*b - c^2)) + 2*(4*a*b^2 - b*c^2)*x)/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c
^4 + (16*a^2*b^3 - 8*a*b^2*c^2 + b*c^4)*x^2 + (16*a^2*b^2*c - 8*a*b*c^3 + c
^5)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(60) = 120$.
time = 0.30, size = 265, normalized size = 3.73

$$-2b\sqrt{-\frac{1}{(4ab - c^{2})^{3}}}\log\left(x + \frac{-32a^{2}b^{3}\sqrt{-\frac{1}{(4ab - c^{2})^{3}}} + 16ab^{2}c^{2}\sqrt{-\frac{1}{(4ab - c^{2})^{3}}} - 2bc^{4}\sqrt{-\frac{1}{(4ab - c^{2})^{3}}} + 2bc}{4b^{2}}\right) + 2b\sqrt{-\frac{1}{(4ab - c^{2})^{3}}}\log\left(x + \frac{32a^{2}b^{3}\sqrt{-\frac{1}{(4ab - c^{2})^{3}}} - 16ab^{2}c^{2}\sqrt{-\frac{1}{(4ab - c^{2})^{3}}} + 2bc^{4}\sqrt{-\frac{1}{(4ab - c^{2})^{3}}} + 2bc}{4b^{2}}\right) + \frac{2bx + c}{4a^{2}b - ac^{2} + x^{2} \cdot (4ab^{2} - bc^{2}) + x\,(4abc - c^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+c*x+a)**2,x)
```

```
[Out] -2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (-32*a**2*b**3*sqrt(-1/(4*a*b - c**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*b - c**2)**3) - 2*b*c**4*sqrt(-1/(4*a*
b - c**2)**3) + 2*b*c)/(4*b**2)) + 2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (
32*a**2*b**3*sqrt(-1/(4*a*b - c**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*b - c
**2)**3) + 2*b*c**4*sqrt(-1/(4*a*b - c**2)**3) + 2*b*c)/(4*b**2)) + (2*b*x
+ c)/(4*a**2*b - a*c**2 + x**2*(4*a*b**2 - b*c**2) + x*(4*a*b*c - c**3))
```

Giac [A]
time = 0.83, size = 67, normalized size = 0.94

$$\frac{4\,b\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{(4\,ab-c^2)^{\frac{3}{2}}}+\frac{2\,bx+c}{(bx^2+cx+a)(4\,ab-c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*b*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/(4*a*b - c^2)^(3/2) + (2*b*x + c)
/((b*x^2 + c*x + a)*(4*a*b - c^2))
```

Mupad [B]
time = 0.17, size = 119, normalized size = 1.68

$$\frac{\frac{c}{4\,a\,b-c^2}+\frac{2\,b\,x}{4\,a\,b-c^2}}{b\,x^2+c\,x+a}-\frac{4\,b\,\mathrm{atan}\left(\frac{\left(\frac{2\,b\,\left(c^3-4\,a\,b\,c\right)}{{\left(4\,a\,b-c^2\right)}^{5/2}}-\frac{4\,b^2\,x}{{\left(4\,a\,b-c^2\right)}^{3/2}}\right)\,\left(4\,a\,b-c^2\right)}{2\,b}\right)}{{\left(4\,a\,b-c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x + b*x^2)^2,x)
```

```
[Out] (c/(4*a*b - c^2) + (2*b*x)/(4*a*b - c^2))/(a + c*x + b*x^2) - (4*b*atan((((
2*b*(c^3 - 4*a*b*c))/(4*a*b - c^2)^(5/2) - (4*b^2*x)/(4*a*b - c^2)^(3/2))*(
4*a*b - c^2))/(2*b)))/(4*a*b - c^2)^(3/2)
```

3.96 $\int \frac{1}{(b+2ax+bx^2)^2}\,dx$

Optimal. Leaf size=72

$$-\frac{a+bx}{2\left(a^2-b^2\right)\left(b+2ax+bx^2\right)}+\frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}$$

```
[Out] 1/2*(-b*x-a)/(a^2-b^2)/(b*x^2+2*a*x+b)+1/2*b*arctanh((b*x+a)/(a^2-b^2)^(1/2
))/(a^2-b^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {628, 632, 212}

$$\frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}-\frac{a+bx}{2\left(a^2-b^2\right)\left(2ax+bx^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] -1/2*(a + b*x)/((a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(a + b*x)/Sqr
t[a^2 - b^2]])/(2*(a^2 - b^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(b+2ax+bx^2)^2}\,dx = -\frac{a+bx}{2(a^2-b^2)(b+2ax+bx^2)} - \frac{b\int \frac{1}{b+2ax+bx^2}\,dx}{2(a^2-b^2)}$$
$$= -\frac{a+bx}{2(a^2-b^2)(b+2ax+bx^2)} + \frac{b\text{Subst}\left(\int \frac{1}{4(a^2-b^2)-x^2}\,dx, x, 2a+2bx\right)}{a^2-b^2}$$
$$= -\frac{a+bx}{2(a^2-b^2)(b+2ax+bx^2)} + \frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 1.00

$$\frac{a+bx}{2(-a^2+b^2)(b+2ax+bx^2)} + \frac{b\tan^{-1}\left(\frac{a+bx}{\sqrt{-a^2+b^2}}\right)}{2(-a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] (a + b*x)/(2*(-a^2 + b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTan[(a + b*x)/Sqrt[-
a^2 + b^2]])/(2*(-a^2 + b^2)^(3/2))
```

Maple [A]
time = 0.58, size = 86, normalized size = 1.19

method	result	size
default	$\frac{2bx+2a}{(-4a^2+4b^2)(b x^2+2ax+b)} + \frac{2b\arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{(-4a^2+4b^2)\sqrt{-a^2+b^2}}$	86
risch	$\frac{-\frac{bx}{4(a^2-b^2)}-\frac{a}{4(a^2-b^2)}}{\frac{1}{2}b x^2+ax+\frac{1}{2}b} + \frac{b\ln\left((-a^2b+b^3)x-(a^2-b^2)^{\frac{3}{2}}-a^3+a b^2\right)}{4(a^2-b^2)^{\frac{3}{2}}} - \frac{b\ln\left((a^2b-b^3)x-(a^2-b^2)^{\frac{3}{2}}+a^3-a b^2\right)}{4(a^2-b^2)^{\frac{3}{2}}}$	150

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+2*a*x+b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*x+2*a)/(-4*a^2+4*b^2)/(b*x^2+2*a*x+b)+2*b/(-4*a^2+4*b^2)/(-a^2+b^2)^(1
/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(64) = 128$.
time = 2.75, size = 317, normalized size = 4.40

$$\left[-\frac{2\,a^{3}-2\,ab^{2}+(b^{2}x^{2}+2\,abx+b^{2})\sqrt{a^{2}-b^{2}}\,\log\left(\frac{b^{2}x^{2}+2\,abx+2\,a^{2}-b^{2}-2\,\sqrt{a^{2}-b^{2}}\,(bx+a)}{bx^{2}+2\,ax+b}\right)+2\,(a^{2}b-b^{3})x}{4\,(a^{4}b-2\,a^{2}b^{3}+b^{5}+(a^{4}b-2\,a^{2}b^{3}+b^{5})x^{2}+2\,(a^{5}-2\,a^{3}b^{2}+ab^{4})x)},\ -\frac{a^{3}-ab^{2}-(b^{2}x^{2}+2\,abx+b^{2})\sqrt{-a^{2}+b^{2}}\,\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\,(bx+a)}{a^{2}-b^{2}}\right)+(a^{2}b-b^{3})x}{2\,(a^{4}b-2\,a^{2}b^{3}+b^{5}+(a^{4}b-2\,a^{2}b^{3}+b^{5})x^{2}+2\,(a^{5}-2\,a^{3}b^{2}+ab^{4})x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^3 - 2*a*b^2 + (b^2*x^2 + 2*a*b*x + b^2)*sqrt(a^2 - b^2)*log((b^2
*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b*x^2 + 2*a*x
+ b)) + 2*(a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 +
b^5)*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x), -1/2*(a^3 - a*b^2 - (b^2*x^2 + 2
*a*b*x + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*x + a)/(a^2 - b^
2)) + (a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)
*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(58) = 116$.
time = 0.30, size = 230, normalized size = 3.19

$$-\frac{b\sqrt{\frac{1}{(a-b)^{3}(a+b)^{3}}}\log\left(x+\frac{-a^{4}b\sqrt{\frac{1}{(a-b)^{3}(a+b)^{3}}}+2a^{2}b^{3}\sqrt{\frac{1}{(a-b)^{3}(a+b)^{3}}}+ab-b^{5}\sqrt{\frac{1}{(a-b)^{3}(a+b)^{3}}}}{b^{2}}\right)}{4}+\frac{b\sqrt{\frac{1}{(a-b)^{3}(a+b)^{3}}}\log\left(x+\frac{a^{4}b\sqrt{\frac{1}{(a-b)^{3}(a+b)^{3}}}-2a^{2}b^{3}\sqrt{\frac{1}{(a-b)^{3}(a+b)^{3}}}+ab+b^{5}\sqrt{\frac{1}{(a-b)^{3}(a+b)^{3}}}}{b^{2}}\right)}{4}+\frac{-a-bx}{2a^{2}b-2b^{3}+x^{2}\cdot(2a^{2}b-2b^{3})+x\,(4a^{3}-4ab^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*sqrt(1/((a - b)**3*(a + b)**3))*log(x + (-a**4*b*sqrt(1/((a - b)**3*(a +
 b)**3)) + 2*a**2*b**3*sqrt(1/((a - b)**3*(a + b)**3)) + a*b - b**5*sqrt(1/
((a - b)**3*(a + b)**3)))/b**2)/4 + b*sqrt(1/((a - b)**3*(a + b)**3))*log(x
 + (a**4*b*sqrt(1/((a - b)**3*(a + b)**3)) - 2*a**2*b**3*sqrt(1/((a - b)**3
```

```
*(a + b)**3)) + a*b + b**5*sqrt(1/((a - b)**3*(a + b)**3)))/b**2)/4 + (-a -
 b*x)/(2*a**2*b - 2*b**3 + x**2*(2*a**2*b - 2*b**3) + x*(4*a**3 - 4*a*b**2)
)
```

Giac [A]
time = 0.71, size = 75, normalized size = 1.04

$$-\frac{b\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{2\,(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{bx+a}{2\,(bx^2+2\,ax+b)(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((b*x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) -
1/2*(b*x + a)/((b*x^2 + 2*a*x + b)*(a^2 - b^2))
```

Mupad [B]
time = 0.32, size = 107, normalized size = 1.49

$$-\frac{\frac{a}{2\,(a^2-b^2)}+\frac{b\,x}{2\,(a^2-b^2)}}{b\,x^2+2\,a\,x+b}+\frac{b\,\mathrm{atan}\left(\frac{-a^3\,1\mathrm{i}-1\mathrm{i}\,x\,a^2\,b+a\,b^2\,1\mathrm{i}+1\mathrm{i}\,x\,b^3}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,1\mathrm{i}}{2\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x + b*x^2)^2,x)
```

```
[Out] (b*atan((a*b^2*1i + b^3*x*1i - a^3*1i - a^2*b*x*1i)/((a + b)^(3/2)*(a - b)^
(3/2)))*1i)/(2*(a + b)^(3/2)*(a - b)^(3/2)) - (a/(2*(a^2 - b^2)) + (b*x)/(2
*(a^2 - b^2)))/(b + 2*a*x + b*x^2)
```

3.97 $\int \frac{1}{(b+2ax-bx^2)^2} \, dx$

Optimal. Leaf size=69

$$-\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

```
[Out] 1/2*(b*x-a)/(a^2+b^2)/(-b*x^2+2*a*x+b)-1/2*b*arctanh((-b*x+a)/(a^2+b^2)^(1/
2))/(a^2+b^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {628, 632, 212}

$$-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x - b*x^2)^(-2),x]
```

```
[Out] -1/2*(a - b*x)/((a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(a - b*x)/Sqr
t[a^2 + b^2]])/(2*(a^2 + b^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(b+2ax-bx^2)^2}\,dx = -\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)} + \frac{b\int \frac{1}{b+2ax-bx^2}\,dx}{2(a^2+b^2)}$$
$$= -\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)} - \frac{b\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2a-2bx\right)}{a^2+b^2}$$
$$= -\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)} - \frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 1.13

$$\frac{\frac{-a+bx}{b+2ax-bx^2} - \frac{b\tan^{-1}\left(\frac{-a+bx}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{2(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x - b*x^2)^(-2),x]
[Out] ((-a + b*x)/(b + 2*a*x - b*x^2) - (b*ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2])/(2*(a^2 + b^2))
```

Maple [A]
time = 0.58, size = 83, normalized size = 1.20

method	result	size
default	$\frac{-2bx+2a}{(-4a^2-4b^2)(-bx^2+2ax+b)} + \frac{2b\,\text{arctanh}\left(\frac{-2bx+2a}{2\sqrt{a^2+b^2}}\right)}{(-4a^2-4b^2)\sqrt{a^2+b^2}}$	83
risch	$\frac{\frac{bx}{4a^2+4b^2} - \frac{a}{4(a^2+b^2)}}{-\frac{1}{2}bx^2+ax+\frac{1}{2}b} + \frac{b\ln\left((a^2b+b^3)x+(a^2+b^2)^{\frac{3}{2}}-a^3-ab^2\right)}{4(a^2+b^2)^{\frac{3}{2}}} - \frac{b\ln\left((-a^2b-b^3)x+(a^2+b^2)^{\frac{3}{2}}+a^3+ab^2\right)}{4(a^2+b^2)^{\frac{3}{2}}}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+2*a*x+b)^2,x,method=_RETURNVERBOSE)
[Out] (-2*b*x+2*a)/(-4*a^2-4*b^2)/(-b*x^2+2*a*x+b)+2*b/(-4*a^2-4*b^2)/(a^2+b^2)^(
1/2)*arctanh(1/2*(-2*b*x+2*a)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.52, size = 97, normalized size = 1.41

$$-\frac{b\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{4\,(a^2+b^2)^{\frac{3}{2}}}+\frac{bx-a}{2\,(a^2b+b^3-(a^2b+b^3)x^2+2\,(a^3+ab^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/(a^2 +
b^2)^(3/2) + 1/2*(b*x - a)/(a^2*b + b^3 - (a^2*b + b^3)*x^2 + 2*(a^3 + a*b^
2)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(65) = 130.
time = 3.39, size = 171, normalized size = 2.48

$$-\frac{2\,a^3+2\,ab^2+(b^2x^2-2\,abx-b^2)\sqrt{a^2+b^2}\,\log\left(\frac{b^2x^2-2\,abx+2\,a^2+b^2+2\,\sqrt{a^2+b^2}\,(bx-a)}{bx^2-2\,ax-b}\right)-2\,(a^2b+b^3)x}{4\,(a^4b+2\,a^2b^3+b^5-(a^4b+2\,a^2b^3+b^5)x^2+2\,(a^5+2\,a^3b^2+ab^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3 + 2*a*b^2 + (b^2*x^2 - 2*a*b*x - b^2)*sqrt(a^2 + b^2)*log((b^2*
x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*x^2 - 2*a*x -
 b)) - 2*(a^2*b + b^3)*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b
^5)*x^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(56) = 112.
time = 0.29, size = 218, normalized size = 3.16

$$-\frac{b\sqrt{\frac{1}{(a^2+b^2)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a^2+b^2)^3}}-2a^2b^3\sqrt{\frac{1}{(a^2+b^2)^3}}-ab-b^5\sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4}+\frac{b\sqrt{\frac{1}{(a^2+b^2)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a^2+b^2)^3}}+2a^2b^3\sqrt{\frac{1}{(a^2+b^2)^3}}-ab+b^5\sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4}+\frac{a-bx}{-2a^2b-2b^3+x^2\cdot(2a^2b+2b^3)+x\,(-4a^3-4ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*sqrt((a**2 + b**2)**(-3))*log(x + (-a**4*b*sqrt((a**2 + b**2)**(-3)) - 2
*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b - b**5*sqrt((a**2 + b**2)**(-3))
)/b**2)/4 + b*sqrt((a**2 + b**2)**(-3))*log(x + (a**4*b*sqrt((a**2 + b**2)*
*(-3)) + 2*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b + b**5*sqrt((a**2 + b*
*2)**(-3)))/b**2)/4 + (a - b*x)/(-2*a**2*b - 2*b**3 + x**2*(2*a**2*b + 2*b*
*3) + x*(-4*a**3 - 4*a*b**2))
```

Giac [A]
time = 0.91, size = 90, normalized size = 1.30

$$-\frac{b\log\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,(a^2+b^2)^{\frac{3}{2}}}-\frac{bx-a}{2\,(bx^2-2\,ax-b)(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^
2 + b^2)))/(a^2 + b^2)^(3/2) - 1/2*(b*x - a)/((b*x^2 - 2*a*x - b)*(a^2 + b^
2))
```

Mupad [B]
time = 0.32, size = 100, normalized size = 1.45

$$-\frac{\frac{a}{2\,(a^2+b^2)}-\frac{b\,x}{2\,(a^2+b^2)}}{-b\,x^2+2\,a\,x+b}+\frac{b\,\mathrm{atan}\left(\frac{a\,b^2\,1\mathrm{i}+a^3\,1\mathrm{i}-b\,x\,(a^2+b^2)\,1\mathrm{i}}{(a^2+b^2)^{3/2}}\right)\,1\mathrm{i}}{2\,(a^2+b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x - b*x^2)^2,x)
```

```
[Out] (b*atan((a*b^2*1i + a^3*1i - b*x*(a^2 + b^2)*1i)/(a^2 + b^2)^(3/2))*1i)/(2*
(a^2 + b^2)^(3/2)) - (a/(2*(a^2 + b^2)) - (b*x)/(2*(a^2 + b^2)))/(b + 2*a*x
 - b*x^2)
```

3.98 $\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx$

Optimal. Leaf size=62

$$-\left(\frac{a}{b}\right)^{-1/n}\tan^{-1}\left(\cot\left(\frac{\pi-2k\pi}{n}\right)-\left(\frac{a}{b}\right)^{-1/n}x\csc\left(\frac{\pi-2k\pi}{n}\right)\right)\csc\left(\frac{\pi-2k\pi}{n}\right)$$

```
[Out] arctan(-cot((-2*Pi*k+Pi)/n)+x*csc((-2*Pi*k+Pi)/n)/((a/b)^(1/n)))*csc((-2*Pi
*k+Pi)/n)/((a/b)^(1/n))
```

Rubi [A]
time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {632, 210}

$$-\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\text{ArcTan}\left(\cot\left(\frac{\pi-2\pi k}{n}\right)-x\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1),x]
```

```
[Out] -((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)]*Csc
[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx = -\left(2\text{Subst}\left(\int \frac{1}{-x^2-4\left(\frac{a}{b}\right)^{2/n}\left(1-\cos^2\left(\frac{\pi-2k\pi}{n}\right)\right)}\,dx, x, 2x-2$$

$$= -\left(\frac{a}{b}\right)^{-1/n}\tan^{-1}\left(\cot\left(\frac{\pi-2k\pi}{n}\right)-\left(\frac{a}{b}\right)^{-1/n}x\csc\left(\frac{\pi-2k\pi}{n}\right.\right.$$

Mathematica [A]
time = 0.07, size = 65, normalized size = 1.05

$$\left(\frac{a}{b}\right)^{-1/n}\tan^{-1}\left(\frac{\left(\left(\frac{a}{b}\right)^{\frac{1}{n}}+x\right)\tan\left(\frac{\pi-2k\pi}{2n}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{n}}-x}\right)\csc\left(\frac{\pi-2k\pi}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1),
x]
```

```
[Out] (ArcTan[(((a/b)^n^(-1) + x)*Tan[(Pi - 2*k*Pi)/(2*n)])/((a/b)^n^(-1) - x)]*C
sc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)
```

Maple [A]
time = 1.62, size = 111, normalized size = 1.79

method	result	size
default	$\frac{\arctan\left(\frac{2x-2\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi(2k-1)}{n}\right)}{2\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}}\left(\cos^2\left(\frac{\pi(2k-1)}{n}\right)\right)+\left(\frac{a}{b}\right)^{\frac{2}{n}}}}\right)}{\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}}\left(\cos^2\left(\frac{\pi(2k-1)}{n}\right)\right)+\left(\frac{a}{b}\right)^{\frac{2}{n}}}}$	111
risch	Expression too large to display	1343

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*Pi*k+Pi)/n)),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/(-((a/b)^(1/n))^2*cos(Pi*(2*k-1)/n)^2+(a/b)^(2/n))^(1/2)*arctan(1/2*(2*x-
2*(a/b)^(1/n)*cos(Pi*(2*k-1)/n))/(-((a/b)^(1/n))^2*cos(Pi*(2*k-1)/n)^2+(a/b
)^(2/n))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(1>0)', see `assume?` for more detai
ls)Is 1
```

Fricas [A]
time = 2.08, size = 89, normalized size = 1.44

$$-\frac{\arctan\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\,\pi k}{n}-\frac{\pi}{n}\right)-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\,\pi k}{n}-\frac{\pi}{n}\right)}\right)}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\,\pi k}{n}-\frac{\pi}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="fricas")
```

```
[Out] -arctan(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x)/((a/b)^(1/n)*sin(2*pi*k/n -
pi/n)))/((a/b)^(1/n)*sin(2*pi*k/n - pi/n))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(46) = 92$.
time = 0.43, size = 212, normalized size = 3.42

$$-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^{2}\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^{2}\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^{2}\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)**(2/n)+x**2-2*(a/b)**(1/n)*x*cos((-2*pi*k+pi)/n)),x)
```

```
[Out] -sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*log(x - (a/b)**(1/n)*c
os(2*pi*k/n - pi/n) - sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*(
-2*(a/b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2 + sqrt(1/((a
/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*log(x - (a/b)**(1/n)*cos(2*pi*k/n
 - pi/n) + sqrt(1/((a/b)**(2/n)*(cos(pi*(2*k - 1)/n)**2 - 1)))*(-2*(a/b)**(
2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2
```

Giac [A]
time = 0.82, size = 100, normalized size = 1.61

$$\frac{\arctan\left(-\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(-\frac{2\,\pi k}{n}+\frac{\pi}{n}\right)-x}{\sqrt{-\cos\left(\frac{2\,\pi k}{n}-\frac{\pi}{n}\right)^{2}+1}\,\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}\right)}{\sqrt{-\cos\left(\frac{2\,\pi k}{n}-\frac{\pi}{n}\right)^{2}+1}\,\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algori
thm="giac")
```

```
[Out] arctan(-((a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x)/(sqrt(-cos(2*pi*k/n - pi/n)
^2 + 1)*(a/b)^(1/n)))/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/b)^(1/n))
```

Mupad [B]
time = 0.25, size = 110, normalized size = 1.77

$$-\frac{\operatorname{atanh}\left(\frac{x-\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)\,\left(\frac{a}{b}\right)^{1/n}}{\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)-1}\,\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)+1}\,\left(\frac{a}{b}\right)^{1/n}}\right)}{\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)-1}\,\sqrt{\cos\left(\frac{\Pi\,(2\,k-1)}{n}\right)+1}\,\left(\frac{a}{b}\right)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/b)^(2/n) + x^2 - 2*x*cos((Pi - 2*Pi*k)/n)*(a/b)^(1/n)),x)
```

```
[Out] -atanh((x - cos((Pi*(2*k - 1))/n)*(a/b)^(1/n))/((cos((Pi*(2*k - 1))/n) - 1)
^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n)))/((cos((Pi*(2*k - 1))
/n) - 1)^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n))
```

3.99 $\int \frac{1}{ab+\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx$

Optimal. Leaf size=33

$$\frac{2\tanh^{-1}\left(\frac{-\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

```
[Out] 2*arctanh((2*b^2*x-(-4*a*b^3+b^2)^(1/2))/b)/b
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.76, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {630, 31}

$$\frac{\log\left(-\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b}-\frac{\log\left(\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] -(Log[b + Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b - Sqrt[b^2 - 4*a*b^3] +
 2*b^2*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{ab+\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx = -\left(b\int \frac{1}{\frac{1}{2}\left(-b+\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\right)+b\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ab^3}\right)-}$$

$$=-\frac{\log\left(b+\sqrt{b^2-4ab^3}-2b^2x\right)}{b}+\frac{\log\left(b-\sqrt{b^2-4ab^3}+2b^2x\right)}{b}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{-\sqrt{-b^2(-1+4ab)}+2b^2x}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(-Sqrt[-(b^2*(-1 + 4*a*b))] + 2*b^2*x)/b])/b
```

Maple [A]
time = 0.60, size = 31, normalized size = 0.94

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{-2b^2x+\sqrt{-b^2\left(4ab-1\right)}}{b}\right)}{b}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*arctanh((-2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

Maxima [A]
time = 0.27, size = 55, normalized size = 1.67

$$-\frac{\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((2*b^2*x - b - sqrt(-4*a*b^3 + b^2))/(2*b^2*x + b - sqrt(-4*a*b^3 + b^
2)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.
time = 1.26, size = 63, normalized size = 1.91

$$\frac{\log\left(\frac{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] (log((2*b^2*x + b - sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b - sqrt(-4*a
*b^3 + b^2))/b))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.
time = 0.13, size = 56, normalized size = 1.70

$$-\frac{\log\left(x-\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b**2*x**2+x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -(log(x - 1/(2*b) - sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) - sq
rt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

Giac [A]
time = 1.05, size = 56, normalized size = 1.70

$$-\frac{\log\left(\frac{\left|2\,b^2x-\sqrt{-4\,ab+1}\,|b|-|b|\right|}{\left|2\,b^2x-\sqrt{-4\,ab+1}\,|b|+|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(abs(2*b^2*x - sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x - sqrt(-4*
a*b + 1)*abs(b) + abs(b)))/abs(b)
```

Mupad [B]
time = 0.26, size = 38, normalized size = 1.15

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b^2-4\,a\,b^3}}{\sqrt{b^2}}-\frac{2\,b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*b + x*(b^2 - 4*a*b^3)^(1/2) - b^2*x^2),x)
```

```
[Out] -(2*atanh((b^2 - 4*a*b^3)^(1/2)/(b^2)^(1/2) - (2*b^2*x)/(b^2)^(1/2)))/(b^2)
^(1/2)
```

3.100 $\int \frac{1}{ab-\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx$

Optimal. Leaf size=31

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

```
[Out] 2*arctanh((2*b^2*x+(-4*a*b^3+b^2)^(1/2))/b)/b
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.87, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {630, 31}

$$\frac{\log\left(\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b}-\frac{\log\left(-\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] -(Log[b - Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b + Sqrt[b^2 - 4*a*b^3] +
 2*b^2*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{ab-\sqrt{b^2-4ab^3}\,x-b^2x^2}\,dx = -\left(b\int \frac{1}{\frac{1}{2}\left(-b-\sqrt{b^2-4ab^3}\right)-b^2x}\,dx\right)+b\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ab^3}\right)-$$

$$= -\frac{\log\left(b-\sqrt{b^2-4ab^3}-2b^2x\right)}{b}+\frac{\log\left(b+\sqrt{b^2-4ab^3}+2b^2x\right)}{b}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-b^2(-1+4ab)}+2b^2x}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
[Out] (2*ArcTanh[(Sqrt[-(b^2*(-1 + 4*a*b))] + 2*b^2*x)/b])/b
```

Maple [A]
time = 0.52, size = 31, normalized size = 1.00

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2b^2x+\sqrt{-b^2\,(4ab-1)}}{b}\right)}{b}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/b*arctanh((2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 1.65

$$-\frac{\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")
[Out] -log((2*b^2*x - b + sqrt(-4*a*b^3 + b^2))/(2*b^2*x + b + sqrt(-4*a*b^3 + b^
2)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.
time = 1.86, size = 59, normalized size = 1.90

$$\frac{\log\left(\frac{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] (log((2*b^2*x + b + sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b + sqrt(-4*a
*b^3 + b^2))/b))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

time = 0.13, size = 56, normalized size = 1.81

$$-\frac{\log\left(x-\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b**2*x**2-x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -(log(x - 1/(2*b) + sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/(2*b) + sq
rt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

Giac [A]

time = 0.88, size = 54, normalized size = 1.74

$$-\frac{\log\left(\frac{\left|2\,b^2x+\sqrt{-4\,ab+1}\,|b|-|b|\right|}{\left|2\,b^2x+\sqrt{-4\,ab+1}\,|b|+|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(abs(2*b^2*x + sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x + sqrt(-4*
a*b + 1)*abs(b) + abs(b)))/abs(b)
```

Mupad [B]

time = 0.13, size = 38, normalized size = 1.23

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b^2-4\,a\,b^3}}{\sqrt{b^2}}+\frac{2\,b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(b^2 - 4*a*b^3)^(1/2) - a*b + b^2*x^2),x)
```

```
[Out] (2*atanh((b^2 - 4*a*b^3)^(1/2)/(b^2)^(1/2) + (2*b^2*x)/(b^2)^(1/2)))/(b^2)^
(1/2)
```

3.101 $\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx$

Optimal. Leaf size=17

$$\tan^{-1}\left(\left(x+\cos\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)$$

```
[Out] arctan((x+cos(1/7))*csc(1/7))*csc(1/7)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {632, 210}

$$\csc\left(\frac{1}{7}\right)\text{ArcTan}\left(\csc\left(\frac{1}{7}\right)\left(x+\cos\left(\frac{1}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

```
[Out] ArcTan[(x + Cos[1/7])*Csc[1/7]]*Csc[1/7]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-x^2-4\sin^2\left(\frac{1}{7}\right)}\,dx, x, 2x+2\cos\left(\frac{1}{7}\right)\right)\right)\\ &= \tan^{-1}\left(\left(x+\cos\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\right)\csc\left(\frac{1}{7}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.12

$$\tan^{-1}\left(\frac{(-1+x)\tan\left(\frac{1}{14}\right)}{1+x}\right)\csc\left(\frac{1}{7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

```
[Out] ArcTan[((-1 + x)*Tan[1/14])/(1 + x)]*Csc[1/7]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(11) = 22.
time = 1.09, size = 33, normalized size = 1.94

method	result	size
default	$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{1}{7}\right)}{2\sqrt{-\left(\cos^{2}\left(\frac{1}{7}\right)\right)+1}}\right)}{\sqrt{-\left(\cos^{2}\left(\frac{1}{7}\right)\right)+1}}$	33
risch	Expression too large to display	3085

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^2+2*x*cos(1/7)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-cos(1/7)^2+1)^(1/2)*arctan(1/2*(2*x+2*cos(1/7))/(-cos(1/7)^2+1)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.
time = 0.47, size = 27, normalized size = 1.59

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="maxima")
```

```
[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

Fricas [A]
time = 0.84, size = 15, normalized size = 0.88

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sin\left(\frac{1}{7}\right)}\right)}{\sin\left(\frac{1}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="fricas")
```

```
[Out] arctan((x + cos(1/7))/sin(1/7))/sin(1/7)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.08, size = 165, normalized size = 9.71

$$-\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\cos^2\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i\cos^2\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**2+2*x*cos(1/7)),x)
```

```
[Out] -I*log(x + cos(1/7) - I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)) + I*cos(1/7
)**2/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos(1/7))*sqrt(co
s(1/7) + 1)) + I*log(x + cos(1/7) - I*cos(1/7)**2/(sqrt(1 - cos(1/7))*sqrt(
cos(1/7) + 1)) + I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos
(1/7))*sqrt(cos(1/7) + 1))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.
time = 1.67, size = 27, normalized size = 1.59

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="giac")
```

```
[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)
```

Mupad [B]
time = 0.12, size = 27, normalized size = 1.59

$$\frac{\operatorname{atan}\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^2}}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x*cos(1/7) + x^2 + 1),x)
[Out] atan((x + cos(1/7))/(1 - cos(1/7)^2)^(1/2))/(1 - cos(1/7)^2)^(1/2)
```

3.102 $\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx$

Optimal. Leaf size=23

$$\tan^{-1}\left(\cot\left(\frac{\pi}{7}\right)+x\csc\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)$$

```
[Out] arctan(cot(1/7*Pi)+x*csc(1/7*Pi))*csc(1/7*Pi)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {632, 210}

$$\csc\left(\frac{\pi}{7}\right)\text{ArcTan}\left(\csc\left(\frac{\pi}{7}\right)\left(x+\cos\left(\frac{\pi}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

```
[Out] ArcTan[(x + Cos[Pi/7])*Csc[Pi/7]]*Csc[Pi/7]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-x^2-4\sin^2\left(\frac{\pi}{7}\right)}\,dx, x, 2x+2\cos\left(\frac{\pi}{7}\right)\right)\right)\\ &= \tan^{-1}\left(\left(x+\cos\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)\right)\csc\left(\frac{\pi}{7}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.

time = 0.03, size = 56, normalized size = 2.43

$$\frac{2\tan^{-1}\left(\frac{\sqrt[7]{-1}-(-1)^{6/7}+2x}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}\right)}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

```
[Out] (2*ArcTan[((-1)^(1/7) - (-1)^(6/7) + 2*x)/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)]
])/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.
time = 0.94, size = 39, normalized size = 1.70

method	result
default	$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{\pi}{7}\right)}{2\sqrt{-\left(\cos^2\left(\frac{\pi}{7}\right)\right)+1}}\right)}{\sqrt{-\left(\cos^2\left(\frac{\pi}{7}\right)\right)+1}}$
norman	$\left(-\frac{4\left(\cos\left(\frac{\pi}{7}\right)+i\sin\left(\frac{\pi}{7}\right)\right)^5}{7}+\frac{\left(\cos\left(\frac{\pi}{7}\right)+i\sin\left(\frac{\pi}{7}\right)\right)^4}{7}-\frac{5\left(\cos\left(\frac{\pi}{7}\right)+i\sin\left(\frac{\pi}{7}\right)\right)^3}{7}+\frac{2\left(\cos\left(\frac{\pi}{7}\right)+i\sin\left(\frac{\pi}{7}\right)\right)^2}{7}-\frac{6\cos\left(\frac{\pi}{7}\right)}{7}-\frac{6i\sin\left(\frac{\pi}{7}\right)}{7}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^2+2*x*cos(1/7*Pi)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-cos(1/7*Pi)^2+1)^(1/2)*arctan(1/2*(2*x+2*cos(1/7*Pi))/(-cos(1/7*Pi)^2+1
)^(1/2))
```

Maxima [A]
time = 0.51, size = 33, normalized size = 1.43

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="maxima")
```

```
[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1)
```

Fricas [A]
time = 1.87, size = 21, normalized size = 0.91

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\,\pi\right)}{\sin\left(\frac{1}{7}\,\pi\right)}\right)}{\sin\left(\frac{1}{7}\,\pi\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="fricas")
```

```
[Out] arctan((x + cos(1/7*pi))/sin(1/7*pi))/sin(1/7*pi)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.31, size = 70, normalized size = 3.04

$$-\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)-\frac{i\left(2-2\cos^2\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}+\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)+\frac{i\left(2-2\cos^2\left(\frac{\pi}{7}\right)\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**2+2*x*cos(1/7*pi)),x)
```

```
[Out] -I*log(x + cos(pi/7) - I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7))
+ I*log(x + cos(pi/7) + I*(2 - 2*cos(pi/7)**2)/(2*sin(pi/7)))/(2*sin(pi/7))
```

Giac [A]
time = 1.60, size = 33, normalized size = 1.43

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\,\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\,\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\,\pi\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="giac")
```

```
[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1)
```

Mupad [B]
time = 0.30, size = 42, normalized size = 1.83

$$-\frac{\operatorname{atanh}\left(\frac{x+\cos\left(\frac{\Pi}{7}\right)}{\sqrt{\cos\left(\frac{\Pi}{7}\right)-1}\,\sqrt{\cos\left(\frac{\Pi}{7}\right)+1}}\right)}{\sqrt{\cos\left(\frac{\Pi}{7}\right)-1}\,\sqrt{\cos\left(\frac{\Pi}{7}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 2*x*cos(Pi/7) + 1),x)
```

```
[Out] -atanh((x + cos(Pi/7))/((cos(Pi/7) - 1)^(1/2)*(cos(Pi/7) + 1)^(1/2)))/((cos
(Pi/7) - 1)^(1/2)*(cos(Pi/7) + 1)^(1/2))
```

3.103 $\int \sqrt{5-6x+9x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2}+\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(-1+3x)\right)$$

```
[Out] 2/3*arcsinh(-1/2+3/2*x)-1/6*(1-3*x)*(9*x^2-6*x+5)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 633, 221}

$$\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] -1/6*((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2]) + (2*ArcSinh[(-1 + 3*x)/2])/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{5-6x+9x^2}\,dx = -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2} + 2\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx$$
$$= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2} + \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{144}}}\,dx, x, -6+18x\right)$$
$$= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2} + \frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(-1+3x)\right)$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.26

$$\frac{1}{6}(-1+3x)\sqrt{5-6x+9x^2} - \frac{2}{3}\log\left(1-3x+\sqrt{5-6x+9x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] ((-1 + 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 - (2*Log[1 - 3*x + Sqrt[5 - 6*x + 9*x^
2]])/3
```

Maple [A]
time = 0.70, size = 29, normalized size = 0.76

method	result	size
default	$\frac{(18x-6)\sqrt{9x^2-6x+5}}{36}+\frac{2\,\mathrm{arcsinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}$	29
risch	$\frac{\sqrt{9x^2-6x+5}\,(3x-1)}{6}+\frac{2\,\mathrm{arcsinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}$	29
trager	$\left(\frac{x}{2}-\frac{1}{6}\right)\sqrt{9x^2-6x+5}+\frac{2\ln\left(-1+3x+\sqrt{9x^2-6x+5}\right)}{3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2-6*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(18*x-6)*(9*x^2-6*x+5)^(1/2)+2/3*arcsinh(3/2*x-1/2)
```

Maxima [A]
time = 0.54, size = 38, normalized size = 1.00

$$\frac{1}{2}\sqrt{9\,x^2-6\,x+5}\,x - \frac{1}{6}\sqrt{9\,x^2-6\,x+5} + \frac{2}{3}\,\mathrm{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 - 6*x + 5)*x - 1/6*sqrt(9*x^2 - 6*x + 5) + 2/3*arcsinh(3/2*x
 - 1/2)
```

Fricas [A]
time = 1.10, size = 40, normalized size = 1.05

$$\frac{1}{6}\sqrt{9\,x^2-6\,x+5}\,(3\,x-1)-\frac{2}{3}\,\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
+ 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{9x^2-6x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2-6*x+5)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 - 6*x + 5), x)
```

Giac [A]
time = 1.29, size = 40, normalized size = 1.05

$$\frac{1}{6}\sqrt{9\,x^2-6\,x+5}\,(3\,x-1)-\frac{2}{3}\,\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
+ 1)
```

Mupad [B]
time = 0.08, size = 39, normalized size = 1.03

$$\frac{2\,\ln\left(3\,x+\sqrt{9\,x^2-6\,x+5}-1\right)}{3}+\left(\frac{x}{2}-\frac{1}{6}\right)\,\sqrt{9\,x^2-6\,x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2 - 6*x + 5)^(1/2),x)
```

```
[Out] (2*log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1))/3 + (x/2 - 1/6)*(9*x^2 - 6*x + 5
)^(1/2)
```

3.104 $\int \sqrt{3-4x-4x^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{4}(1+2x)\sqrt{3-4x-4x^2}+\sin^{-1}\left(\frac{1}{2}+x\right)$$

```
[Out] arcsin(1/2+x)+1/4*(1+2*x)*(-4*x^2-4*x+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 633, 222}

$$\text{ArcSin}\left(x+\frac{1}{2}\right)+\frac{1}{4}\sqrt{-4x^2-4x+3}\,(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{3-4x-4x^2}\,dx = \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} + 2\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx$$
$$= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -4-8x\right)$$
$$= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} + \sin^{-1}\left(\frac{1}{2}+x\right)$$

Mathematica [**A**]
time = 0.06, size = 49, normalized size = 1.63

$$\frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} - 2\tan^{-1}\left(\frac{\sqrt{3-4x-4x^2}}{3+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*x - 4*x^2],x]
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 - 2*ArcTan[Sqrt[3 - 4*x - 4*x^2]/(3 + 2
*x)]
```

Maple [**A**]
time = 0.57, size = 25, normalized size = 0.83

method	result
default	$-\frac{(-8x-4)\sqrt{-4x^2-4x+3}}{16} + \arcsin\left(x+\frac{1}{2}\right)$
risch	$-\frac{(4x^2+4x-3)(2x+1)}{4\sqrt{-4x^2-4x+3}} + \arcsin\left(x+\frac{1}{2}\right)$
trager	$\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{-4x^2-4x+3} + \text{RootOf}\left(\_Z^2+1\right)\ln\left(-2x\,\text{RootOf}\left(\_Z^2+1\right)+\sqrt{-4x^2-4x+3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-4*x+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/16*(-8*x-4)*(-4*x^2-4*x+3)^(1/2)+arcsin(x+1/2)
```

Maxima [**A**]
time = 0.51, size = 38, normalized size = 1.27

$$\frac{1}{2}\sqrt{-4\,x^2-4\,x+3}\,x + \frac{1}{4}\sqrt{-4\,x^2-4\,x+3} - \arcsin\left(-x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 - 4*x + 3)*x + 1/4*sqrt(-4*x^2 - 4*x + 3) - arcsin(-x - 1/2
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.
time = 1.68, size = 53, normalized size = 1.77

$$\frac{1}{4}\sqrt{-4\,x^2-4\,x+3}\,(2\,x+1)-\arctan\left(\frac{\sqrt{-4\,x^2-4\,x+3}\,(2\,x+1)}{4\,x^2+4\,x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) - arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x +
 1)/(4*x^2 + 4*x - 3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-4x^2-4x+3}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(-4*x**2 - 4*x + 3), x)
```

Giac [A]
time = 1.39, size = 24, normalized size = 0.80

$$\frac{1}{4}\sqrt{-4\,x^2-4\,x+3}\,(2\,x+1)+\arcsin\left(x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

Mupad [B]
time = 0.05, size = 23, normalized size = 0.77

$$\operatorname{asin}\left(x+\frac{1}{2}\right)+\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{-4\,x^2-4\,x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 4*x^2 - 4*x)^(1/2),x)
```

```
[Out] asin(x + 1/2) + (x/2 + 1/4)*(3 - 4*x^2 - 4*x)^(1/2)
```

3.105 $\int \sqrt{-8+6x+9x^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2}-\frac{3}{2}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)$$

```
[Out] -3/2*arctanh((1+3*x)/(9*x^2+6*x-8)^(1/2))+1/6*(1+3*x)*(9*x^2+6*x-8)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 635, 212}

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - (3*ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x +
 9*x^2]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \sqrt{-8+6x+9x^2}\,dx = \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} - \frac{9}{2}\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$$
$$= \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} - 9\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)$$
$$= \frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} - \frac{3}{2}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 1.00

$$\frac{1}{6}(1+3x)\sqrt{-8+6x+9x^2} - 3\tanh^{-1}\left(\frac{\sqrt{-8+6x+9x^2}}{-2+3x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - 3*ArcTanh[Sqrt[-8 + 6*x + 9*x^2]/(-2
 + 3*x)]
```

Maple [A]
time = 0.49, size = 50, normalized size = 1.02

method	result	size
trager	$\left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8} - \frac{3\ln\left(\sqrt{9x^2+6x-8}+1+3x\right)}{2}$	40
default	$\frac{(18x+6)\sqrt{9x^2+6x-8}}{36} - \frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{2}$	50
risch	$\frac{(3x+1)\sqrt{9x^2+6x-8}}{6} - \frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2+6*x-8)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(18*x+6)*(9*x^2+6*x-8)^(1/2)-1/2*ln(1/9*(9*x+3)*9^(1/2)+(9*x^2+6*x-8)^
(1/2))*9^(1/2)
```

Maxima [A]
time = 0.54, size = 52, normalized size = 1.06

$$\frac{1}{2}\sqrt{9x^2+6x-8}\,x + \frac{1}{6}\sqrt{9x^2+6x-8} - \frac{3}{2}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 + 6*x - 8)*x + 1/6*sqrt(9*x^2 + 6*x - 8) - 3/2*log(18*x + 6*
sqrt(9*x^2 + 6*x - 8) + 6)
```

Fricas [A]
time = 1.26, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{9x^2+6x-8}\,(3x+1)+\frac{3}{2}\log\left(-3x+\sqrt{9x^2+6x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(-3*x + sqrt(9*x^2 + 6*x - 8)
- 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{9x^2+6x-8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 + 6*x - 8), x)
```

Giac [A]
time = 1.46, size = 41, normalized size = 0.84

$$\frac{1}{6}\sqrt{9x^2+6x-8}\,(3x+1)+\frac{3}{2}\log\left(\left|-3x+\sqrt{9x^2+6x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x -
 8) - 1))
```

Mupad [B]
time = 0.21, size = 39, normalized size = 0.80

$$\left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8}-\frac{3\ln\left(3x+\sqrt{9x^2+6x-8}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x + 9*x^2 - 8)^(1/2),x)
```

```
[Out] (x/2 + 1/6)*(6*x + 9*x^2 - 8)^(1/2) - (3*log(3*x + (6*x + 9*x^2 - 8)^(1/2)
+ 1))/2
```

3.106 $\int \sqrt{2+4x+3x^2}\, dx$

Optimal. Leaf size=45

$$\frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}$$

```
[Out] 1/9*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)+1/6*(2+3*x)*(3*x^2+4*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 633, 221}

$$\frac{1}{6}\sqrt{3x^2+4x+2}\,(3x+2) + \frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3*Sqrt[3]
)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{2+4x+3x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{1}{3}\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx$$

$$= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}}\,dx, x, 4+6x\right)}{6\sqrt{6}}$$

$$= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Mathematica [A]
time = 0.08, size = 53, normalized size = 1.18

$$\frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} - \frac{\log\left(-2-3x+\sqrt{6+12x+9x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x + 3*x^2],x]
[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 - Log[-2 - 3*x + Sqrt[6 + 12*x + 9*x^2]
]/(3*Sqrt[3])
```

Maple [A]
time = 0.61, size = 35, normalized size = 0.78

method	result
default	$\frac{(6x+4)\sqrt{3x^2+4x+2}}{12} + \frac{\sqrt{3}\ \text{arcsinh}\left(\frac{3\sqrt{2}\,\left(x+\frac{2}{3}\right)}{2}\right)}{9}$
risch	$\frac{(2+3x)\sqrt{3x^2+4x+2}}{6} + \frac{\sqrt{3}\ \text{arcsinh}\left(\frac{3\sqrt{2}\,\left(x+\frac{2}{3}\right)}{2}\right)}{9}$
trager	$\left(\frac{1}{3}+\frac{x}{2}\right)\sqrt{3x^2+4x+2} + \frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(3\,\text{RootOf}\left(\_Z^2-3\right)x+3\sqrt{3x^2+4x+2}+2\,\text{RootOf}\left(\_Z^2-3\right)\right)}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*(6*x+4)*(3*x^2+4*x+2)^(1/2)+1/9*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Maxima [A]
time = 0.51, size = 46, normalized size = 1.02

$$\frac{1}{2}\sqrt{3x^2+4x+2}\,x+\frac{1}{9}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}\,(3x+2)\right)+\frac{1}{3}\sqrt{3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 4*x + 2)*x + 1/9*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2)) +
1/3*sqrt(3*x^2 + 4*x + 2)
```

Fricas [A]
time = 1.55, size = 58, normalized size = 1.29

$$\frac{1}{6}\sqrt{3x^2+4x+2}\,(3x+2)+\frac{1}{18}\sqrt{3}\,\log\left(-\sqrt{3}\sqrt{3x^2+4x+2}\,(3x+2)-9x^2-12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) + 1/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
+ 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3x^2+4x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 4*x + 2), x)
```

Giac [A]
time = 1.01, size = 53, normalized size = 1.18

$$\frac{1}{6}\sqrt{3x^2+4x+2}\,(3x+2)-\frac{1}{9}\sqrt{3}\,\log\left(-\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3x^2+4x+2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x -
 sqrt(3*x^2 + 4*x + 2)) - 2)
```

Mupad [B]
time = 0.19, size = 48, normalized size = 1.07

$$\frac{\sqrt{3}\ \ln\left(\sqrt{3\,x^2+4\,x+2}+\frac{\sqrt{3}\,(3\,x+2)}{3}\right)}{9}+\left(\frac{x}{2}+\frac{1}{3}\right)\,\sqrt{3\,x^2+4\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*log((4*x + 3*x^2 + 2)^(1/2) + (3^(1/2)*(3*x + 2))/3))/9 + (x/2 + 1
/3)*(4*x + 3*x^2 + 2)^(1/2)
```

3.107 $\int \sqrt{2+4x-3x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2}-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

```
[Out] -5/9*arcsin(1/10*(2-3*x)*10^(1/2))*3^(1/2)-1/6*(2-3*x)*(-3*x^2+4*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {626, 633, 222}

$$-\frac{5\text{ArcSin}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}-\frac{1}{6}\sqrt{-3x^2+4x+2}\,(2-3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] -1/6*((2 - 3*x)*Sqrt[2 + 4*x - 3*x^2]) - (5*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*
Sqrt[3])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{2+4x-3x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} + \frac{5}{3}\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx$$
$$= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} - \frac{1}{6}\sqrt{\frac{5}{6}}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}}\,dx, x, 4-6x\right)$$
$$= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} - \frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Mathematica [A]
time = 0.12, size = 61, normalized size = 1.36

$$\frac{1}{6}(-2+3x)\sqrt{2+4x-3x^2} + \frac{10\tan^{-1}\left(\frac{-2-\sqrt{10}+3x}{\sqrt{6+12x-9x^2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 4*x - 3*x^2],x]
[Out] ((-2 + 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 + (10*ArcTan[(-2 - Sqrt[10] + 3*x)/Sqr
t[6 + 12*x - 9*x^2]])/(3*Sqrt[3])
```

Maple [A]
time = 0.53, size = 35, normalized size = 0.78

method	result
default	$-\frac{(-6x+4)\sqrt{-3x^2+4x+2}}{12} + \frac{5\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\,\left(x-\frac{2}{3}\right)}{10}\right)}{9}$
risch	$-\frac{(3x^2-4x-2)(-2+3x)}{6\sqrt{-3x^2+4x+2}} + \frac{5\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\,\left(x-\frac{2}{3}\right)}{10}\right)}{9}$
trager	$\left(-\frac{1}{3}+\frac{x}{2}\right)\sqrt{-3x^2+4x+2} + \frac{5\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-3x\,\mathrm{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x+2}+2\,\mathrm{RootOf}(\_\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(-6*x+4)*(-3*x^2+4*x+2)^(1/2)+5/9*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3
))
```

Maxima [A]
time = 0.54, size = 46, normalized size = 1.02

$$\frac{1}{2}\sqrt{-3x^2+4x+2}\,x-\frac{5}{9}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}\,(3x-2)\right)-\frac{1}{3}\sqrt{-3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-3*x^2 + 4*x + 2)*x - 5/9*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
 - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Fricas [A]
time = 1.17, size = 60, normalized size = 1.33

$$\frac{1}{6}\sqrt{-3x^2+4x+2}\,(3x-2)-\frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+4x+2}\,(3x-2)}{3\,(3x^2-4x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(
-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4*x - 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-3x^2+4x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x+2)**(1/2),x)
[Out] Integral(sqrt(-3*x**2 + 4*x + 2), x)
```

Giac [A]
time = 0.75, size = 36, normalized size = 0.80

$$\frac{1}{6}\sqrt{-3x^2+4x+2}\,(3x-2)+\frac{5}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}\,(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3*
x - 2))
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.78

$$\frac{5\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{10}\,(3x-2)}{10}\right)}{9}+\left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 3*x^2 + 2)^(1/2),x)
```

```
[Out] (5*3^(1/2)*asin((10^(1/2)*(3*x - 2))/10))/9 + (x/2 - 1/3)*(4*x - 3*x^2 + 2)
^(1/2)
```

3.108 $\int \sqrt{2+5x+3x^2}\,dx$

Optimal. Leaf size=62

$$\frac{1}{12}(5+6x)\sqrt{2+5x+3x^2}-\frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{24\sqrt{3}}$$

```
[Out] -1/72*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x+2)^(1/2))*3^(1/2)+1/12*(5+6*x)
*(3*x^2+5*x+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 635, 212}

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2
+ 5*x + 3*x^2])]/(24*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \sqrt{2+5x+3x^2}\,dx = \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{1}{24}\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$$
$$= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{1}{12}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{2+5x+3x^2}}\right)$$
$$= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{24\sqrt{3}}$$

Mathematica [A]
time = 0.12, size = 56, normalized size = 0.90

$$\frac{1}{36}\left(3(5+6x)\sqrt{2+5x+3x^2} - \sqrt{3}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}+\frac{5x}{3}+x^2}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] (3*(5 + 6*x)*Sqrt[2 + 5*x + 3*x^2] - Sqrt[3]*ArcTanh[Sqrt[2/3 + (5*x)/3 + x
^2]/(1 + x)])/36
```

Maple [A]
time = 0.47, size = 50, normalized size = 0.81

method	result
default	$\frac{(5+6x)\sqrt{3x^2+5x+2}}{12} - \frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{72}$
risch	$\frac{(5+6x)\sqrt{3x^2+5x+2}}{12} - \frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{72}$
trager	$\left(\frac{5}{12}+\frac{x}{2}\right)\sqrt{3x^2+5x+2} - \frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\text{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2+5x+2}+5\,\text{RootOf}\left(\_Z^2-3\right)\right.}{72}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(5+6*x)*(3*x^2+5*x+2)^(1/2)-1/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2
)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.54, size = 58, normalized size = 0.94

$$\frac{1}{2}\sqrt{3x^2+5x+2}\,x-\frac{1}{72}\sqrt{3}\,\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\frac{5}{12}\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 5*x + 2)*x - 1/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x +
 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x + 2)
```

Fricas [A]
time = 1.41, size = 58, normalized size = 0.94

$$\frac{1}{12}\sqrt{3x^2+5x+2}\,(6x+5)+\frac{1}{144}\sqrt{3}\,\log\left(-4\sqrt{3}\sqrt{3x^2+5x+2}\,(6x+5)+72x^2+120x+49\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3*
x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x + 49)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3x^2+5x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 5*x + 2), x)
```

Giac [A]
time = 0.57, size = 54, normalized size = 0.87

$$\frac{1}{12}\sqrt{3x^2+5x+2}\,(6x+5)+\frac{1}{72}\sqrt{3}\,\log\left(\left|-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3x^2+5x+2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sqr
t(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

Mupad [B]
time = 0.20, size = 48, normalized size = 0.77

$$\left(\frac{x}{2}+\frac{5}{12}\right)\,\sqrt{3\,x^2+5\,x+2}-\frac{\sqrt{3}\,\ln\left(\sqrt{3\,x^2+5\,x+2}+\frac{\sqrt{3}\,\left(3\,x+\frac{5}{2}\right)}{3}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3*x^2 + 2)^(1/2),x)
```

```
[Out] (x/2 + 5/12)*(5*x + 3*x^2 + 2)^(1/2) - (3^(1/2)*log((5*x + 3*x^2 + 2)^(1/2)
 + (3^(1/2)*(3*x + 5/2))/3))/72
```

3.109 $\int \sqrt{2+5x-3x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2}-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

```
[Out] 49/72*arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 633, 222}

$$-\frac{49\text{ArcSin}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x+2}\,(5-6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -1/12*((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2]) - (49*ArcSin[(5 - 6*x)/7])/(24*Sqrt
[3])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{2+5x-3x^2}\,dx = -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} + \frac{49}{24}\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx$$

$$= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} - \frac{7\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}}\,dx, x, 5-6x\right)}{24\sqrt{3}}$$

$$= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} - \frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Mathematica [A]
time = 0.09, size = 56, normalized size = 1.30

$$\frac{1}{36}\left(3(-5+6x)\sqrt{2+5x-3x^2} - 49\sqrt{3}\tan^{-1}\left(\frac{\sqrt{6+15x-9x^2}}{1+3x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 5*x - 3*x^2],x]
[Out] (3*(-5 + 6*x)*Sqrt[2 + 5*x - 3*x^2] - 49*Sqrt[3]*ArcTan[Sqrt[6 + 15*x - 9*x
^2]/(1 + 3*x)])/36
```

Maple [A]
time = 0.51, size = 32, normalized size = 0.74

method	result
default	$\frac{49\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{72} - \frac{(5-6x)\sqrt{-3x^2+5x+2}}{12}$
risch	$-\frac{(3x^2-5x-2)(-5+6x)}{12\sqrt{-3x^2+5x+2}} + \frac{49\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{72}$
trager	$\left(-\frac{5}{12}+\frac{x}{2}\right)\sqrt{-3x^2+5x+2} + \frac{49\,\text{RootOf}\left(\_Z^2+3\right)\ln\left(-6x\,\text{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x+2}+5\,\text{RootOf}\right.}{72}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 49/72*arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)
```

Maxima [A]
time = 0.53, size = 41, normalized size = 0.95

$$\frac{1}{2}\sqrt{-3x^2+5x+2}\,x - \frac{49}{72}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right) - \frac{5}{12}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 5*x + 2)*x - 49/72*sqrt(3)*arcsin(-6/7*x + 5/7) - 5/12*sq
rt(-3*x^2 + 5*x + 2)
```

Fricas [A]
time = 1.02, size = 60, normalized size = 1.40

$$\frac{1}{12}\,\sqrt{-3\,x^2+5\,x+2}\,(6\,x-5)-\frac{49}{72}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{-3\,x^2+5\,x+2}\,(6\,x-5)}{6\,(3\,x^2-5\,x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) - 49/72*sqrt(3)*arctan(1/6*sqrt(3)*sq
rt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5*x - 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2+5x+2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 5*x + 2), x)
```

Giac [A]
time = 0.52, size = 31, normalized size = 0.72

$$\frac{1}{12}\,\sqrt{-3\,x^2+5\,x+2}\,(6\,x-5)+\frac{49}{72}\,\sqrt{3}\,\arcsin\left(\frac{6}{7}\,x-\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*x - 5/7)
```

Mupad [B]
time = 0.15, size = 30, normalized size = 0.70

$$\frac{49\,\sqrt{3}\,\mathrm{asin}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{72}+\left(\frac{x}{2}-\frac{5}{12}\right)\,\sqrt{-3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 3*x^2 + 2)^(1/2),x)
```

```
[Out] (49*3^(1/2)*asin((6*x)/7 - 5/7))/72 + (x/2 - 5/12)*(5*x - 3*x^2 + 2)^(1/2)
```

3.110 $\int \sqrt{-2+4x+3x^2}\,dx$

Optimal. Leaf size=59

$$\frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2}-\frac{5\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}$$

```
[Out] -5/9*arctanh(1/3*(2+3*x)*3^(1/2)/(3*x^2+4*x-2)^(1/2))*3^(1/2)+1/6*(2+3*x)*(
3*x^2+4*x-2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 635, 212}

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-
2 + 4*x + 3*x^2])])/(3*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \sqrt{-2+4x+3x^2}\,dx = \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{5}{3}\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx$$
$$= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{10}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{4+6x}{\sqrt{-2+4x+3x^2}}\right)$$
$$= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{5\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}$$

Mathematica [A]
time = 0.17, size = 61, normalized size = 1.03

$$\frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{10\tanh^{-1}\left(\frac{\sqrt{-6+12x+9x^2}}{2+\sqrt{10}+3x}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 4*x + 3*x^2],x]
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (10*ArcTanh[Sqrt[-6 + 12*x + 9*x^2]/
(2 + Sqrt[10] + 3*x)])/(3*Sqrt[3])
```

Maple [A]
time = 0.55, size = 50, normalized size = 0.85

method	result
default	$\frac{(6x+4)\sqrt{3x^2+4x-2}}{12} - \frac{5\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{9}$
risch	$\frac{(2+3x)\sqrt{3x^2+4x-2}}{6} - \frac{5\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{9}$
trager	$\left(\frac{1}{3}+\frac{x}{2}\right)\sqrt{3x^2+4x-2} - \frac{5\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(3\,\text{RootOf}\left(\_Z^2-3\right)x+3\sqrt{3x^2+4x-2}+2\,\text{RootOf}\left(\_Z^2-3\right)\right)}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*(6*x+4)*(3*x^2+4*x-2)^(1/2)-5/9*ln(1/3*(2+3*x)*3^(1/2)+(3*x^2+4*x-2)^(
1/2))*3^(1/2)
```

Maxima [A]
time = 0.51, size = 58, normalized size = 0.98

$$\frac{1}{2}\sqrt{3x^2+4x-2}\,x - \frac{5}{9}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right) + \frac{1}{3}\sqrt{3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 4*x - 2)*x - 5/9*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x -
2) + 6*x + 4) + 1/3*sqrt(3*x^2 + 4*x - 2)
```

Fricas [A]
time = 2.58, size = 58, normalized size = 0.98

$$\frac{1}{6}\sqrt{3\,x^2+4\,x-2}\,(3\,x+2)+\frac{5}{18}\sqrt{3}\,\log\left(-\sqrt{3}\,\sqrt{3\,x^2+4\,x-2}\,(3\,x+2)+9\,x^2+12\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2
+ 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3x^2+4x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 4*x - 2), x)
```

Giac [A]
time = 0.54, size = 54, normalized size = 0.92

$$\frac{1}{6}\sqrt{3\,x^2+4\,x-2}\,(3\,x+2)+\frac{5}{9}\sqrt{3}\,\log\left(\left|-\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2+4\,x-2}\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)
*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

Mupad [B]
time = 0.19, size = 48, normalized size = 0.81

$$\left(\frac{x}{2}+\frac{1}{3}\right)\,\sqrt{3\,x^2+4\,x-2}-\frac{5\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2+4\,x-2}+\frac{\sqrt{3}\,(3\,x+2)}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2 - 2)^(1/2),x)
```

```
[Out] (x/2 + 1/3)*(4*x + 3*x^2 - 2)^(1/2) - (5*3^(1/2)*log((4*x + 3*x^2 - 2)^(1/2
) + (3^(1/2)*(3*x + 2))/3))/9
```

3.111 $\int \sqrt{-2+4x-3x^2}\,dx$

Optimal. Leaf size=59

$$-\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2}+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{3\sqrt{3}}$$

```
[Out] 1/9*arctan(1/3*(2-3*x)*3^(1/2)/(-3*x^2+4*x-2)^(1/2))*3^(1/2)-1/6*(2-3*x)*(-
3*x^2+4*x-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 635, 210}

$$\frac{\text{ArcTan}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -1/6*((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2]) + ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2
 + 4*x - 3*x^2])]/(3*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \sqrt{-2+4x-3x^2}\,dx = -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} - \frac{1}{3}\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx$$
$$= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, \frac{4-6x}{\sqrt{-2+4x-3x^2}}\right)$$
$$= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} + \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{3\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 59, normalized size = 1.00

$$\frac{1}{6}(-2+3x)\sqrt{-2+4x-3x^2} - \frac{i\log\left(2i-3ix+\sqrt{-6+12x-9x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] ((-2 + 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 - ((I/3)*Log[2*I - (3*I)*x + Sqrt[-6
+ 12*x - 9*x^2]])/Sqrt[3]
```

Maple [A]
time = 0.79, size = 46, normalized size = 0.78

method	result
default	$-\frac{(-6x+4)\sqrt{-3x^2+4x-2}}{12} - \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x-\frac{2}{3})}{\sqrt{-3x^2+4x-2}}\right)}{9}$
risch	$-\frac{(3x^2-4x+2)(-2+3x)}{6\sqrt{-3x^2+4x-2}} - \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x-\frac{2}{3})}{\sqrt{-3x^2+4x-2}}\right)}{9}$
trager	$\left(-\frac{1}{3}+\frac{x}{2}\right)\sqrt{-3x^2+4x-2} + \frac{\text{RootOf}\left(\_Z^2+3\right)\ln\left(3x\,\text{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x-2}-2\,\text{RootOf}\left(\_Z\right.\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-6*x+4)*(-3*x^2+4*x-2)^(1/2)-1/9*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*
x^2+4*x-2)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 46, normalized size = 0.78

$$\frac{1}{2}\sqrt{-3x^2+4x-2}\,x+\frac{1}{9}i\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}\,(3x-2)\right)-\frac{1}{3}\sqrt{-3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 4*x - 2)*x + 1/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
 - 1/3*sqrt(-3*x^2 + 4*x - 2)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.45, size = 86, normalized size = 1.46

$$\frac{1}{6}\sqrt{-3x^2+4x-2}\,(3x-2)-\frac{1}{18}i\sqrt{3}\log\left(-\frac{2\left(i\sqrt{3}\sqrt{-3x^2+4x-2}+3x-2\right)}{x}\right)+\frac{1}{18}i\sqrt{3}\log\left(-\frac{2\left(-i\sqrt{3}\sqrt{-3x^2+4x-2}+3x-2\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x - 2)*(3*x - 2) - 1/18*I*sqrt(3)*log(-2*(I*sqrt(3)*sqr
t(-3*x^2 + 4*x - 2) + 3*x - 2)/x) + 1/18*I*sqrt(3)*log(-2*(-I*sqrt(3)*sqrt(
-3*x^2 + 4*x - 2) + 3*x - 2)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-3x^2+4x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 4*x - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*x^2 + 4*x - 2), x)
```

Mupad [B]
time = 0.05, size = 36, normalized size = 0.61

$$\frac{\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{2}\,(3\,x-2)\,1\mathrm{i}}{2}\right)}{9}+\left(\frac{x}{2}-\frac{1}{3}\right)\,\sqrt{-3\,x^2+4\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 3*x^2 - 2)^(1/2),x)
```

```
[Out] (3^(1/2)*asin((2^(1/2)*(3*x - 2)*1i)/2))/9 + (x/2 - 1/3)*(4*x - 3*x^2 - 2)^
(1/2)
```

3.112 $\int \sqrt{-2+5x+3x^2}\,dx$

Optimal. Leaf size=62

$$\frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{24\sqrt{3}}$$

```
[Out] -49/72*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x-2)^(1/2))*3^(1/2)+1/12*(5+6*x
)*(3*x^2+5*x-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 635, 212}

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sq
rt[-2 + 5*x + 3*x^2])])/(24*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-2+5x+3x^2}\,dx &= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49}{24}\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx\\ &= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49}{12}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{5+6x}{\sqrt{-2+5x+3x^2}}\right)\\ &= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2}-\frac{49\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{24\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 56, normalized size = 0.90

$$\frac{1}{36}\left(3(5+6x)\sqrt{-2+5x+3x^2}-49\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{-\frac{2}{3}+\frac{5x}{3}+x^2}}{2+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] (3*(5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2] - 49*Sqrt[3]*ArcTanh[Sqrt[-2/3 + (5*x)/
3 + x^2]/(2 + x)])/36
```

Maple [A]
time = 0.50, size = 50, normalized size = 0.81

method	result
default	$\frac{(5+6x)\sqrt{3x^2+5x-2}}{12}-\frac{49\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{72}$
risch	$\frac{(5+6x)\sqrt{3x^2+5x-2}}{12}-\frac{49\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{72}$
trager	$\left(\frac{5}{12}+\frac{x}{2}\right)\sqrt{3x^2+5x-2}+\frac{49\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\text{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2+5x-2}-5\,\text{RootOf}\left(\_Z^2\right.\right.}{72}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(5+6*x)*(3*x^2+5*x-2)^(1/2)-49/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-
2)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.53, size = 58, normalized size = 0.94

$$\frac{1}{2}\sqrt{3x^2+5x-2}\,x-\frac{49}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right)+\frac{5}{12}\sqrt{3x^2+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 5*x - 2)*x - 49/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x
- 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x - 2)
```

Fricas [A]
time = 2.65, size = 58, normalized size = 0.94

$$\frac{1}{12}\sqrt{3x^2+5x-2}\,(6x+5)+\frac{49}{144}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2+5x-2}\,(6x+5)+72x^2+120x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3
*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3x^2+5x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 5*x - 2), x)
```

Giac [A]
time = 0.73, size = 54, normalized size = 0.87

$$\frac{1}{12}\sqrt{3x^2+5x-2}\,(6x+5)+\frac{49}{72}\sqrt{3}\log\left(\left|-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3x^2+5x-2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

Mupad [B]
time = 0.22, size = 48, normalized size = 0.77

$$\left(\frac{x}{2}+\frac{5}{12}\right)\,\sqrt{3\,x^2+5\,x-2}-\frac{49\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2+5\,x-2}+\frac{\sqrt{3}\,\left(3\,x+\frac{5}{2}\right)}{3}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3*x^2 - 2)^(1/2),x)
```

```
[Out] (x/2 + 5/12)*(5*x + 3*x^2 - 2)^(1/2) - (49*3^(1/2)*log((5*x + 3*x^2 - 2)^(1
/2) + (3^(1/2)*(3*x + 5/2))/3))/72
```

3.113 $\int \sqrt{-2+5x-3x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2}-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

```
[Out] 1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {626, 633, 222}

$$-\frac{\text{ArcSin}(5-6x)}{24\sqrt{3}}-\frac{1}{12}\sqrt{-3x^2+5x-2}\,(5-6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -1/12*((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2]) - ArcSin[5 - 6*x]/(24*Sqrt[3])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \sqrt{-2+5x-3x^2}\,dx = -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2} + \frac{1}{24}\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx$$
$$= -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-6x\right)}{24\sqrt{3}}$$
$$= -\frac{1}{12}(5-6x)\sqrt{-2+5x-3x^2} - \frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Mathematica [**A**]
time = 0.09, size = 56, normalized size = 1.44

$$\frac{1}{36}\left(3(-5+6x)\sqrt{-2+5x-3x^2} - \sqrt{3}\tan^{-1}\left(\frac{\sqrt{-6+15x-9x^2}}{-2+3x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + 5*x - 3*x^2],x]
[Out] (3*(-5 + 6*x)*Sqrt[-2 + 5*x - 3*x^2] - Sqrt[3]*ArcTan[Sqrt[-6 + 15*x - 9*x^
2]/(-2 + 3*x)])/36
```

Maple [**A**]
time = 0.52, size = 32, normalized size = 0.82

method	result
default	$\frac{\arcsin(-5+6x)\sqrt{3}}{72} - \frac{(5-6x)\sqrt{-3x^2+5x-2}}{12}$
risch	$-\frac{(3x^2-5x+2)(-5+6x)}{12\sqrt{-3x^2+5x-2}} + \frac{\arcsin(-5+6x)\sqrt{3}}{72}$
trager	$\left(-\frac{5}{12}+\frac{x}{2}\right)\sqrt{-3x^2+5x-2} - \frac{\text{RootOf}\left(\_Z^2+3\right)\ln\left(6x\,\text{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x-2}-5\,\text{RootOf}\left(\_\right.\right.}{72}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)
```

Maxima [**A**]
time = 0.53, size = 41, normalized size = 1.05

$$\frac{1}{2}\sqrt{-3\,x^2+5\,x-2}\,x + \frac{1}{72}\sqrt{3}\arcsin\left(6\,x-5\right) - \frac{5}{12}\sqrt{-3\,x^2+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 5*x - 2)*x + 1/72*sqrt(3)*arcsin(6*x - 5) - 5/12*sqrt(-3*
x^2 + 5*x - 2)
```

Fricas [A]
time = 2.52, size = 60, normalized size = 1.54

$$\frac{1}{12}\,\sqrt{-3\,x^2+5\,x-2}\,(6\,x-5)-\frac{1}{72}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{-3\,x^2+5\,x-2}\,(6\,x-5)}{6\,(3\,x^2-5\,x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) - 1/72*sqrt(3)*arctan(1/6*sqrt(3)*sqr
t(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5*x + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-3x^2+5x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 5*x - 2), x)
```

Giac [A]
time = 1.62, size = 31, normalized size = 0.79

$$\frac{1}{12}\,\sqrt{-3\,x^2+5\,x-2}\,(6\,x-5)+\frac{1}{72}\,\sqrt{3}\,\arcsin\left(6\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)
```

Mupad [B]
time = 0.05, size = 30, normalized size = 0.77

$$\frac{\sqrt{3}\,\mathrm{asin}(6\,x-5)}{72}+\left(\frac{x}{2}-\frac{5}{12}\right)\,\sqrt{-3\,x^2+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 3*x^2 - 2)^(1/2),x)
```

```
[Out] (3^(1/2)*asin(6*x - 5))/72 + (x/2 - 5/12)*(5*x - 3*x^2 - 2)^(1/2)
```

3.114 $\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(-1+3x)\right)$$

```
[Out] 1/3*arcsinh(-1/2+3/2*x)
```

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {633, 221}

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] ArcSinh[(-1 + 3*x)/2]/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx &= \frac{1}{36}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{144}}}\,dx, x, -6+18x\right)\\ &= \frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(-1+3x)\right)\end{aligned}$$

Mathematica [A]

time = 0.05, size = 24, normalized size = 1.71

$$-\frac{1}{3}\log\left(1-3x+\sqrt{5-6x+9x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] -1/3*Log[1 - 3*x + Sqrt[5 - 6*x + 9*x^2]]
```

Maple [A]
time = 0.51, size = 9, normalized size = 0.64

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{3x}{2}-\frac{1}{2}\right)}{3}$	9
trager	$-\frac{\ln\left(\sqrt{9x^{2}-6x+5}+1-3x\right)}{3}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2-6*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arcsinh(3/2*x-1/2)
```

Maxima [A]
time = 0.48, size = 8, normalized size = 0.57

$$\frac{1}{3} \operatorname{arsinh}\left(\frac{3}{2}x - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsinh(3/2*x - 1/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.
time = 2.70, size = 20, normalized size = 1.43

$$-\frac{1}{3} \log\left(-3\,x + \sqrt{9\,x^{2} - 6\,x + 5} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{9x^{2} - 6x + 5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2-6*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 - 6*x + 5), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(8) = 16$.
time = 1.51, size = 40, normalized size = 2.86

$$\frac{1}{6}\sqrt{9\,x^2-6\,x+5}\,(3\,x-1)-\frac{2}{3}\,\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5)
+ 1)
```

Mupad [B]
time = 0.20, size = 20, normalized size = 1.43

$$\frac{\ln\left(3\,x+\sqrt{9\,x^2-6\,x+5}-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2 - 6*x + 5)^(1/2),x)
```

```
[Out] log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1)/3
```

3.115 $\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin^{-1}\left(\frac{1}{2}+x\right)$$

```
[Out] 1/2*arcsin(1/2+x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\frac{1}{2}\text{ArcSin}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] ArcSin[1/2 + x]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx &= -\left(\frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -4-8x\right)\right)\\ &= \frac{1}{2}\sin^{-1}\left(\frac{1}{2}+x\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

time = 0.05, size = 25, normalized size = 2.50

$$-\tan^{-1}\left(\frac{\sqrt{3-4x-4x^2}}{3+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] -ArcTan[Sqrt[3 - 4*x - 4*x^2]/(3 + 2*x)]
```

Maple [A]
time = 0.57, size = 7, normalized size = 0.70

method	result	size
default	$\frac{\arcsin\left(x+\frac{1}{2}\right)}{2}$	7
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(2x\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-4x^2-4x+3}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2-4*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(x+1/2)
```

Maxima [A]
time = 0.50, size = 8, normalized size = 0.80

$$-\frac{1}{2}\arcsin\left(-x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-x - 1/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(6) = 12.
time = 1.98, size = 33, normalized size = 3.30

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-4\,x^2-4\,x+3}\,(2\,x+1)}{4\,x^2+4\,x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x + 1)/(4*x^2 + 4*x - 3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^2 - 4x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**2 - 4*x + 3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(6) = 12.
time = 1.27, size = 24, normalized size = 2.40

$$\frac{1}{4}\sqrt{-4\,x^2 - 4\,x + 3}\,(2\,x + 1) + \arcsin\left(x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

Mupad [B]
time = 0.13, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(x + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 4*x^2 - 4*x)^(1/2),x)
```

```
[Out] asin(x + 1/2)/2
```

3.116 $\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)$$

```
[Out] 1/3*arctanh((1+3*x)/(9*x^2+6*x-8)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {635, 212}

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)\\ &= \frac{1}{3}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 24, normalized size = 0.96

$$-\frac{1}{3}\log\left(-1-3x+\sqrt{-8+6x+9x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] -1/3*Log[-1 - 3*x + Sqrt[-8 + 6*x + 9*x^2]]
```

Maple [A]
time = 0.53, size = 30, normalized size = 1.20

method	result	size
trager	$\frac{\ln\left(\sqrt{9x^2+6x-8}+1+3x\right)}{3}$	21
default	$\frac{\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)\sqrt{9}}{9}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+6*x-8)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*ln(1/9*(9*x+3)*9^(1/2)+(9*x^2+6*x-8)^(1/2))*9^(1/2)
```

Maxima [A]
time = 0.49, size = 22, normalized size = 0.88

$$\frac{1}{3}\log\left(18\,x+6\,\sqrt{9\,x^2+6\,x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

Fricas [A]
time = 2.22, size = 20, normalized size = 0.80

$$-\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{9x^2+6x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 6*x - 8), x)
```

Giac [A]
time = 1.54, size = 41, normalized size = 1.64

$$\frac{1}{6}\sqrt{9x^2+6x-8}\,(3x+1)+\frac{3}{2}\log\left(\left|-3x+\sqrt{9x^2+6x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x -
 8) - 1))
```

Mupad [B]
time = 0.22, size = 20, normalized size = 0.80

$$\frac{\ln\left(3x+\sqrt{9x^2+6x-8}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x + 9*x^2 - 8)^(1/2),x)
```

```
[Out] log(3*x + (6*x + 9*x^2 - 8)^(1/2) + 1)/3
```

3.117 $\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 221}

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}}\,dx, x, 4+6x\right)}{2\sqrt{6}}\\ &= \frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 1.50

$$-\frac{\log \left(-2-3x+\sqrt{6+12x+9x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] -(Log[-2 - 3*x + Sqrt[6 + 12*x + 9*x^2]]/Sqrt[3])
```

Maple [A]
time = 0.62, size = 15, normalized size = 0.83

method	result	size
default	$\frac{\sqrt{3}\, \operatorname{arcsinh}\left(\frac{3\sqrt{2}\,\left(x+\frac{2}{3}\right)}{2}\right)}{3}$	15
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right) \ln\left(-3 \operatorname{RootOf}\left(\_Z^2-3\right) x+3\sqrt{3x^2+4x+2}-2 \operatorname{RootOf}\left(\_Z^2-3\right)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Maxima [A]
time = 0.54, size = 16, normalized size = 0.89

$$\frac{1}{3}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{2}\,(3\,x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.
time = 2.26, size = 38, normalized size = 2.11

$$\frac{1}{6}\,\sqrt{3}\,\log\left(-\sqrt{3}\,\sqrt{3\,x^2+4\,x+2}\,(3\,x+2)-9\,x^2-12\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2+4x+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 4*x + 2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(16) = 32$.
time = 1.73, size = 53, normalized size = 2.94

$$\frac{1}{6}\sqrt{3\,x^2+4\,x+2}\,(3\,x+2)-\frac{1}{9}\sqrt{3}\,\log\left(-\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2+4\,x+2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x -
 sqrt(3*x^2 + 4*x + 2)) - 2)
```

Mupad [**B**]
time = 0.22, size = 26, normalized size = 1.44

$$\frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x+\frac{2}{3}\right)+\sqrt{3\,x^2+4\,x+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 + 2)^(1/2)))/3
```

3.118 $\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arcsin(1/10*(2-3*x)*10^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$-\frac{\text{ArcSin}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}}\,dx, x, 4-6x\right)}{2\sqrt{30}}\\ &= -\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.
time = 0.07, size = 39, normalized size = 2.05

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{2}-\sqrt{2+4x-3x^2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x - 3*x^2],x]
[Out] (-2*ArcTan[(Sqrt[3]*x)/(Sqrt[2] - Sqrt[2 + 4*x - 3*x^2])])/Sqrt[3]
```

Maple [A]
time = 0.53, size = 15, normalized size = 0.79

method	result	size
default	$\frac{\sqrt{3}\,\arcsin\left(\frac{3\sqrt{10}\,\left(x-\frac{2}{3}\right)}{10}\right)}{3}$	15
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(3x\,\mathrm{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x+2}-2\,\mathrm{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3))
```

Maxima [A]
time = 0.50, size = 16, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\,\arcsin\left(-\frac{1}{10}\sqrt{10}\,(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.
time = 1.84, size = 40, normalized size = 2.11

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{-3\,x^2+4\,x+2}\,(3\,x-2)}{3\,(3\,x^2-4\,x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4
*x - 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2 + 4x + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 4*x + 2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.
time = 1.32, size = 36, normalized size = 1.89

$$\frac{1}{6}\,\sqrt{-3\,x^2+4\,x+2}\,(3\,x-2)+\frac{5}{9}\,\sqrt{3}\,\arcsin\left(\frac{1}{10}\,\sqrt{10}\,(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sqrt(10)*(3*
x - 2))
```

Mupad [B]
time = 0.14, size = 16, normalized size = 0.84

$$\frac{\sqrt{3}\,\mathrm{asin}\left(\frac{\sqrt{40}\,(6\,x-4)}{40}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*asin((40^(1/2)*(6*x - 4))/40))/3
```

3.119 $\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}}$$

[Out] 1/3*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x+2)^(1/2))*3^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {635, 212}

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 5*x + 3*x^2],x]

[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{2+5x+3x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 30, normalized size = 0.86

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}+\frac{5x}{3}+x^2}}{1+x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x + 3*x^2],x]
[Out] (2*ArcTanh[Sqrt[2/3 + (5*x)/3 + x^2]/(1 + x)])/Sqrt[3]
```

Maple [A]
time = 0.46, size = 30, normalized size = 0.86

method	result	size
default	$\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{3}$	30
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2+5x+2}-5\,\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.52, size = 28, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3\,x^2+5\,x+2}+6\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)
```

Fricas [A]
time = 1.74, size = 38, normalized size = 1.09

$$\frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2+5\,x+2}\,(6\,x+5)+72\,x^2+120\,x+49\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x
+ 49)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2 + 5x + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 5*x + 2), x)
```

Giac [A]
time = 1.16, size = 54, normalized size = 1.54

$$\frac{1}{12}\sqrt{3\,x^2+5\,x+2}\,(6\,x+5)+\frac{1}{72}\sqrt{3}\,\log\left(\left|-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2+5\,x+2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sqr
t(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

Mupad [B]
time = 0.24, size = 26, normalized size = 0.74

$$\frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x+\frac{5}{6}\right)+\sqrt{3\,x^2+5\,x+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 + 2)^(1/2)))/3
```

3.120 $\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

```
[Out] 1/3*arcsin(-5/7+6/7*x)*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$-\frac{\text{ArcSin}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}}\,dx, x, 5-6x\right)}{7\sqrt{3}}\\ &= -\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 30, normalized size = 1.76

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{6+15x-9x^2}}{1+3x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x - 3*x^2],x]
[Out] (-2*ArcTan[Sqrt[6 + 15*x - 9*x^2]/(1 + 3*x)])/Sqrt[3]
```

Maple [A]
time = 0.49, size = 12, normalized size = 0.71

method	result	size
default	$\frac{\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)\sqrt{3}}{3}$	12
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(6x\operatorname{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x+2}-5\operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsin(-5/7+6/7*x)*3^(1/2)
```

Maxima [A]
time = 0.53, size = 11, normalized size = 0.65

$$-\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arcsin(-6/7*x + 5/7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.
time = 1.27, size = 40, normalized size = 2.35

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}(6x-5)}{6(3x^2-5x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5
*x - 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2+5x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 5*x + 2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.
time = 0.90, size = 31, normalized size = 1.82

$$\frac{1}{12}\sqrt{-3\,x^2+5\,x+2}\,(6\,x-5)+\frac{49}{72}\sqrt{3}\,\arcsin\left(\frac{6}{7}\,x-\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*x - 5/7)
```

Mupad [B]
time = 0.17, size = 11, normalized size = 0.65

$$\frac{\sqrt{3}\,\mathrm{asin}\left(\frac{6\,x}{7}-\frac{5}{7}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x - 3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*asin((6*x)/7 - 5/7))/3
```

3.121 $\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(1/3*(2+3*x)*3^(1/2)/(3*x^2+4*x-2)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {635, 212}

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{4+6x}{\sqrt{-2+4x+3x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 0.84

$$-\frac{\log \left(-2-3x+\sqrt{-6+12x+9x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x + 3*x^2],x]
[Out] -(Log[-2 - 3*x + Sqrt[-6 + 12*x + 9*x^2]]/Sqrt[3])
```

Maple [A]
time = 0.51, size = 30, normalized size = 0.94

method	result	size
default	$\frac{\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)\sqrt{3}}{3}$	30
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(3\,\mathrm{RootOf}\left(\_Z^2-3\right)x+3\sqrt{3x^2+4x-2}+2\,\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(1/3*(2+3*x)*3^(1/2)+(3*x^2+4*x-2)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.51, size = 28, normalized size = 0.88

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4)
```

Fricas [A]
time = 1.84, size = 37, normalized size = 1.16

$$\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\sqrt{3x^2+4x-2}(3x+2)+9x^2+12x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+4*x-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**2 + 4*x - 2), x)
```

Giac [A]
time = 0.68, size = 54, normalized size = 1.69

$$\frac{1}{6}\sqrt{3x^2+4x-2}\,(3x+2)+\frac{5}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+4x-2}\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)
*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

Mupad [B]
time = 0.23, size = 26, normalized size = 0.81

$$\frac{\sqrt{3}\ln\left(\sqrt{3}\left(x+\frac{2}{3}\right)+\sqrt{3x^2+4x-2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 3*x^2 - 2)^(1/2),x)
[Out] (3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 - 2)^(1/2)))/3
```

3.122 $\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(1/3*(2-3*x)*3^(1/2)/(-3*x^2+4*x-2)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {635, 210}

$$-\frac{\text{ArcTan}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -(ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx = 2\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, \frac{4-6x}{\sqrt{-2+4x-3x^2}}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 33, normalized size = 1.00

$$\frac{i\log\left(2-3x-i\sqrt{-6+12x-9x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x - 3*x^2],x]
[Out] (I*Log[2 - 3*x - I*Sqrt[-6 + 12*x - 9*x^2]])/Sqrt[3]
```

Maple [A]
time = 0.62, size = 26, normalized size = 0.79

method	result	size
default	$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x-\frac{2}{3}\right)}{\sqrt{-3x^2+4x-2}}\right)}{3}$	26
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(-3x\operatorname{RootOf}\left(\_Z^2+3\right)+3\sqrt{-3x^2+4x-2}+2\operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 16, normalized size = 0.48

$$-\frac{1}{3}i\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")
[Out] -1/3*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.54, size = 67, normalized size = 2.03

$$\frac{1}{6}i\sqrt{3}\log\left(-\frac{2\left(i\sqrt{3}\sqrt{-3x^2+4x-2}+3x-2\right)}{x}\right)-\frac{1}{6}i\sqrt{3}\log\left(-\frac{2\left(-i\sqrt{3}\sqrt{-3x^2+4x-2}+3x-2\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*I*sqrt(3)*log(-2*(I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) + 3*x - 2)/x) - 1/6*
I*sqrt(3)*log(-2*(-I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) + 3*x - 2)/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 4*x - 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^2 + 4*x - 2), x)
```

Mupad [**B**]
time = 0.14, size = 17, normalized size = 0.52

$$-\frac{\sqrt{3}\,\mathrm{asin}\left(\sqrt{2}\,\left(\frac{3\,x}{2}-1\right)\,1{}\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 3*x^2 - 2)^(1/2),x)
```

```
[Out] -(3^(1/2)*asin(2^(1/2)*((3*x)/2 - 1)*1i))/3
```

3.123 $\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x-2)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {635, 212}

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx = 2\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{5+6x}{\sqrt{-2+5x+3x^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.07, size = 30, normalized size = 0.86

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-\frac{2}{3}+\frac{5x}{3}+x^2}}{2+x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x + 3*x^2],x]
[Out] (2*ArcTanh[Sqrt[-2/3 + (5*x)/3 + x^2]/(2 + x)])/Sqrt[3]
```

Maple [A]
time = 0.48, size = 30, normalized size = 0.86

method	result	size
default	$\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x-2}\right)\sqrt{3}}{3}$	30
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2+5x-2}-5\,\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-2)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.54, size = 28, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5)
```

Fricas [A]
time = 1.70, size = 38, normalized size = 1.09

$$\frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2+5x-2}\,(6x+5)+72x^2+120x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x
+ 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2+5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 5*x - 2), x)
```

Giac [A]
time = 0.64, size = 54, normalized size = 1.54

$$\frac{1}{12}\sqrt{3\,x^2+5\,x-2}\,(6\,x+5)+\frac{49}{72}\sqrt{3}\,\log\left(\left|-2\,\sqrt{3}\,\left(\sqrt{3}\,x-\sqrt{3\,x^2+5\,x-2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

Mupad [B]
time = 0.23, size = 26, normalized size = 0.74

$$\frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x+\frac{5}{6}\right)+\sqrt{3\,x^2+5\,x-2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 - 2)^(1/2),x)
```

```
[Out] (3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 - 2)^(1/2)))/3
```

3.124 $\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx$

Optimal. Leaf size=13

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

[Out] 1/3*arcsin(-5+6*x)*3^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$-\frac{\text{ArcSin}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-2 + 5*x - 3*x^2],x]

[Out] -(ArcSin[5 - 6*x]/Sqrt[3])

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-6x\right)}{\sqrt{3}}\\ &= -\frac{\sin^{-1}(5-6x)}{\sqrt{3}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

time = 0.05, size = 30, normalized size = 2.31

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-6+15x-9x^2}}{-2+3x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] (-2*ArcTan[Sqrt[-6 + 15*x - 9*x^2]/(-2 + 3*x)])/Sqrt[3]
```

Maple [A]
time = 0.62, size = 12, normalized size = 0.92

method	result	size
default	$\frac{\arcsin(-5+6x)\sqrt{3}}{3}$	12
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-6x\,\mathrm{RootOf}\left(\_Z^2+3\right)+6\sqrt{-3x^2+5x-2}+5\,\mathrm{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arcsin(-5+6*x)*3^(1/2)
```

Maxima [A]
time = 0.49, size = 11, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arcsin\left(6\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x - 5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.
time = 1.65, size = 40, normalized size = 3.08

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+5\,x-2}\,(6\,x-5)}{6\,(3\,x^2-5\,x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5
*x + 2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2+5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 5*x - 2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.
time = 0.66, size = 31, normalized size = 2.38

$$\frac{1}{12}\sqrt{-3\,x^2+5\,x-2}\,(6\,x-5)+\frac{1}{72}\,\sqrt{3}\,\arcsin\left(6\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)
```

Mupad [**B**]
time = 0.16, size = 11, normalized size = 0.85

$$\frac{\sqrt{3}\,\mathrm{asin}(6\,x-5)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x - 3*x^2 - 2)^(1/2),x)
```

```
[Out] (3^(1/2)*asin(6*x - 5))/3
```

3.125 $\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] arcsinh(1/2*(2*c*x+b)/c^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {633, 221}

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

```
[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4c}}}\,dx, x, b+2cx\right)}{2c}$$

$$= \frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Mathematica [A]
time = 0.14, size = 44, normalized size = 2.00

$$-\frac{\log\left(b+2cx-\sqrt{c}\sqrt{4+\frac{b^2}{c}+4bx+4cx^2}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

```
[Out] -(Log[b + 2*c*x - Sqrt[c]*Sqrt[4 + b^2/c + 4*b*x + 4*c*x^2]]/Sqrt[c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(16) = 32$.
time = 0.68, size = 51, normalized size = 2.32

method	result	size
default	$\frac{\ln\left(\frac{(4cx+2b)\sqrt{4}}{4\sqrt{c}}+\sqrt{\frac{b^2+4c}{c}+4bx+4c\,x^2}\right)\sqrt{4}}{2\sqrt{c}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1/4*(4*c*x+2*b)*4^(1/2)/c^(1/2)+((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2))*4
^(1/2)/c^(1/2)
```

Maxima [A]
time = 0.26, size = 16, normalized size = 0.73

$$\frac{\operatorname{arsinh}\left(\frac{2\,cx+b}{2\,\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/2*(2*c*x + b)/sqrt(c))/sqrt(c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(16) = 32$.
time = 2.12, size = 137, normalized size = 6.23

$$\left[\frac{\log\left(-4\,c^2x^2 - 4\,bcx - b^2 - (2\,cx + b)\sqrt{c}\sqrt{\frac{4\,c^2x^2 + 4\,bcx + b^2 + 4\,c}{c}} - 2\,c\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{(2\,cx+b)\sqrt{-c}\sqrt{\frac{4\,c^2x^2 + 4\,bcx + b^2 + 4\,c}{c}}}{4\,c^2x^2+4\,bcx+b^2+4\,c}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-4*c^2*x^2 - 4*b*c*x - b^2 - (2*c*x + b)*sqrt(c)*sqrt((4*c^2*x^2 +
 4*b*c*x + b^2 + 4*c)/c) - 2*c)/sqrt(c), -sqrt(-c)*arctan((2*c*x + b)*sqrt(
-c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c)/(4*c^2*x^2 + 4*b*c*x + b^2 +
4*c))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{1}{\sqrt{\frac{b^2}{c} + 4bx + 4cx^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b**2+4*c)/c+4*b*x+4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(b**2/c + 4*b*x + 4*c*x**2 + 4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(16) = 32$.
time = 0.75, size = 60, normalized size = 2.73

$$-\frac{\log\left(\left|-bc^2 - \left(2\,\sqrt{c^3}\,x - \sqrt{4\,c^3x^2 + 4\,bc^2x + b^2c + 4\,c^2}\right)\sqrt{c}\,|c|\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-b*c^2 - (2*sqrt(c^3)*x - sqrt(4*c^3*x^2 + 4*b*c^2*x + b^2*c + 4*c
^2))*sqrt(c)*abs(c)))/sqrt(c)
```

Mupad [B]
time = 0.42, size = 40, normalized size = 1.82

$$\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+\sqrt{\frac{b^2+4\,c}{c}+4\,b\,x+4\,c\,x^2}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2),x)
[Out] log((b + 2*c*x)/c^(1/2) + ((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2))/c^(1/2)
```

3.126 $\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -arcsin(1/2*(-2*c*x+b)/c^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {633, 222}

$$-\frac{\text{ArcSin}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4c}}}\,dx, x, b-2cx\right)}{2c}$$

$$= -\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 123 vs. $2(23) = 46$.
time = 0.22, size = 123, normalized size = 5.35

$$2\left(-\frac{\tan^{-1}\left(\frac{2\sqrt{-c^2}\,x-\sqrt{c}\sqrt{4-\frac{b^2}{c}+4bx-4cx^2}}{b}\right)}{2\sqrt{c}}-\frac{\log\left(2c^2x^2+c\left(-1-bx+\sqrt{-c}\,x\sqrt{4-\frac{b^2}{c}+4bx-4cx^2}\right)\right)}{4\sqrt{-c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] 2*(-1/2*ArcTan[(2*Sqrt[-c^2]*x - Sqrt[c]*Sqrt[4 - b^2/c + 4*b*x - 4*c*x^2])
/b]/Sqrt[c] - Log[2*c^2*x^2 + c*(-1 - b*x + Sqrt[-c]*x*Sqrt[4 - b^2/c + 4*b
*x - 4*c*x^2])]/(4*Sqrt[-c]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.
time = 0.60, size = 44, normalized size = 1.91

method	result	size
default	$\frac{\arctan\left(\frac{2\sqrt{c}\left(x-\frac{b}{2c}\right)}{\sqrt{-4c\,x^2+4bx-\frac{b^2-4c}{c}}}\right)}{\sqrt{c}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-4*c)/c)^(1/2))
```

Maxima [A]
time = 0.52, size = 19, normalized size = 0.83

$$-\frac{\arcsin\left(-\frac{2\,cx-b}{2\,\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/2*(2*c*x - b)/sqrt(c))/sqrt(c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(19) = 38.
time = 1.46, size = 141, normalized size = 6.13

$$\left[-\frac{\sqrt{-c}\,\log\left(4\,c^2x^2-4\,bcx+b^2-(2\,cx-b)\sqrt{-c}\,\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}-2\,c\right)}{2\,c},-\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\,\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}}{4\,c^2x^2-4\,bcx+b^2-4\,c}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-c)*log(4*c^2*x^2 - 4*b*c*x + b^2 - (2*c*x - b)*sqrt(-c)*sqrt(-(
4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c) - 2*c)/c, -arctan((2*c*x - b)*sqrt(c)*s
qrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)
)/sqrt(c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{1}{\sqrt{-\frac{b^2}{c}+4bx-4cx^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+4*c)/c+4*b*x-4*c*x**2)**(1/2),x)
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(19) = 38.
time = 1.55, size = 65, normalized size = 2.83

$$\frac{\sqrt{-c}\,\log\left(b\sqrt{-c}\,c-\left(2\,\sqrt{-c^3}\,x-\sqrt{-4\,c^3x^2+4\,bc^2x-b^2c+4\,c^2}\right)|c|\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-c)*log(b*sqrt(-c)*c - (2*sqrt(-c^3)*x - sqrt(-4*c^3*x^2 + 4*b*c^2*x -
 b^2*c + 4*c^2))*abs(c))/c
```

Mupad [B]
time = 0.41, size = 46, normalized size = 2.00

$$\frac{\ln\left(\frac{b-2\,c\,x}{\sqrt{-c}}+\sqrt{4\,b\,x+\frac{4\,c-b^2}{c}-4\,c\,x^2}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2),x)
```

```
[Out] log((b - 2*c*x)/(-c)^(1/2) + (4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2))/(-c)^
(1/2)
```

3.127 $\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -arcsin((-2*c*x+b)/c^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {633, 222}

$$-\frac{\text{ArcSin}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c}}}\,dx, x, b-2cx\right)}{c}$$

$$= -\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(20) = 40.
time = 0.21, size = 123, normalized size = 6.15

$$-\frac{-2\sqrt{-c}\,\tan^{-1}\left(\frac{\sqrt{c}\left(-2\sqrt{-c}\,x+\sqrt{1-\frac{b^2}{c}+4bx-4cx^2}\right)}{b}\right)+\sqrt{c}\,\log\left(c\left(-1-4bx+8cx^2+4\sqrt{-c}\,x\sqrt{1-\frac{b^2}{c}+4bx-4cx^2}\right)\right)}{2\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -1/2*(-2*Sqrt[-c]*ArcTan[(Sqrt[c]*(-2*Sqrt[-c]*x + Sqrt[1 - b^2/c + 4*b*x -
 4*c*x^2]))/b] + Sqrt[c]*Log[c*(-1 - 4*b*x + 8*c*x^2 + 4*Sqrt[-c]*x*Sqrt[1
- b^2/c + 4*b*x - 4*c*x^2])])/Sqrt[-c^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(16) = 32.
time = 0.60, size = 44, normalized size = 2.20

method	result	size
default	$\frac{\arctan\left(\frac{2\sqrt{c}\,\left(x-\frac{b}{2c}\right)}{\sqrt{-4c\,x^2+4bx-\frac{b^2-c}{c}}}\right)}{\sqrt{c}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-c)/c)^(1/2))
```

Maxima [A]
time = 0.48, size = 19, normalized size = 0.95

$$-\frac{\arcsin\left(-\frac{2\,cx-b}{\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*c*x - b)/sqrt(c))/sqrt(c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(19) = 38.
time = 1.94, size = 143, normalized size = 7.15

$$\left[-\frac{\sqrt{-c}\,\log\left(8\,c^2x^2-8\,bcx+2\,b^2-2\,(2\,cx-b)\sqrt{-c}\,\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}-c\right)}{2\,c},-\frac{\arctan\left(\frac{(2\,cx-b)\sqrt{c}\,\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}}{4\,c^2x^2-4\,bcx+b^2-c}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + 2*b^2 - 2*(2*c*x - b)*sqrt(-c)*sqr
t(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c) - c)/c, -arctan((2*c*x - b)*sqrt(c)*s
qrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - c))/sq
rt(c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{1}{\sqrt{-\frac{b^2}{c}+4bx-4cx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(19) = 38.
time = 1.65, size = 63, normalized size = 3.15

$$\frac{\sqrt{-c}\,\log\left(b\sqrt{-c}\,c-\left(2\,\sqrt{-c^3}\,x-\sqrt{-4\,c^3x^2+4\,bc^2x-b^2c+c^2}\right)|c|\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-c)*log(b*sqrt(-c)*c - (2*sqrt(-c^3)*x - sqrt(-4*c^3*x^2 + 4*b*c^2*x -
 b^2*c + c^2))*abs(c))/c
```

Mupad [B]
time = 0.40, size = 44, normalized size = 2.20

$$\frac{\ln\left(\frac{b-2\,c\,x}{\sqrt{-c}}+\sqrt{\frac{c-b^2}{c}+4\,b\,x-4\,c\,x^2}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2),x)
```

```
[Out] log((b - 2*c*x)/(-c)^(1/2) + ((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2))/(-c)^(1
/2)
```

3.128 $\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

```
[Out] -2*(3+2*x)/(x^2+3*x+2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {627}

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + x^2)^(-3/2),x]
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx = -\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

Mathematica [A]
time = 0.08, size = 19, normalized size = 1.00

$$-\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + x^2)^(-3/2),x]
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Maple [A]
time = 0.46, size = 18, normalized size = 0.95

method	result	size
default	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
trager	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
risch	$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$	18
gosper	$-\frac{2(2+x)(x+1)(2x+3)}{(x^2+3x+2)^{\frac{3}{2}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(2*x+3)/(x^2+3*x+2)^(1/2)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 1.37

$$-\frac{4\,x}{\sqrt{x^2+3\,x+2}}-\frac{6}{\sqrt{x^2+3\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*x/sqrt(x^2 + 3*x + 2) - 6/sqrt(x^2 + 3*x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.
time = 1.33, size = 38, normalized size = 2.00

$$-\frac{2\left(2\,x^2+\sqrt{x^2+3\,x+2}\,(2\,x+3)+6\,x+4\right)}{x^2+3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*x^2 + sqrt(x^2 + 3*x + 2)*(2*x + 3) + 6*x + 4)/(x^2 + 3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(x^2+3x+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3*x+2)**(3/2),x)
```

```
[Out] Integral((x**2 + 3*x + 2)**(-3/2), x)
```

Giac [A]
time = 1.38, size = 17, normalized size = 0.89

$$-\frac{2\,(2\,x+3)}{\sqrt{x^2+3\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*x + 3)/sqrt(x^2 + 3*x + 2)
```

Mupad [B]
time = 0.05, size = 15, normalized size = 0.79

$$-\frac{4\,\left(x+\frac{3}{2}\right)}{\sqrt{x^2+3\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + x^2 + 2)^(3/2),x)
```

```
[Out] -(4*(x + 3/2))/(3*x + x^2 + 2)^(1/2)
```

3.129 $\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{3-x}{9\sqrt{27-24x+4x^2}}$$

```
[Out] 1/9*(3-x)/(4*x^2-24*x+27)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{627\}$

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 24*x + 4*x^2)^(-3/2),x]
```

```
[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(27-24x+4x^2)^{3/2}}\,dx = \frac{3-x}{9\sqrt{27-24x+4x^2}}$$

Mathematica [A]
time = 0.09, size = 23, normalized size = 1.00

$$\frac{3-x}{9\sqrt{27-24x+4x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 24*x + 4*x^2)^(-3/2),x]
```

```
[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Maple [A]
time = 0.52, size = 20, normalized size = 0.87

method	result	size
trager	$-\frac{x-3}{9\sqrt{4x^2-24x+27}}$	18
risch	$-\frac{x-3}{9\sqrt{4x^2-24x+27}}$	18
default	$-\frac{8x-24}{72\sqrt{4x^2-24x+27}}$	20
gosper	$-\frac{(-3+2x)(2x-9)(x-3)}{9(4x^2-24x+27)^{\frac{3}{2}}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-24*x+27)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(8*x-24)/(4*x^2-24*x+27)^(1/2)
```

Maxima [A]
time = 0.29, size = 30, normalized size = 1.30

$$-\frac{x}{9\sqrt{4x^2-24x+27}}+\frac{1}{3\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*x/sqrt(4*x^2 - 24*x + 27) + 1/3/sqrt(4*x^2 - 24*x + 27)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(17) = 34.
time = 2.18, size = 41, normalized size = 1.78

$$-\frac{4x^2+2\sqrt{4x^2-24x+27}(x-3)-24x+27}{18(4x^2-24x+27)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18*(4*x^2 + 2*sqrt(4*x^2 - 24*x + 27)*(x - 3) - 24*x + 27)/(4*x^2 - 24*x
 + 27)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(4x^2-24x+27)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-24*x+27)**(3/2),x)
[Out] Integral((4*x**2 - 24*x + 27)**(-3/2), x)
```

Giac [A]
time = 1.39, size = 17, normalized size = 0.74

$$-\frac{x-3}{9\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="giac")
[Out] -1/9*(x - 3)/sqrt(4*x^2 - 24*x + 27)
```

Mupad [B]
time = 0.06, size = 17, normalized size = 0.74

$$-\frac{x-3}{9\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 24*x + 27)^(3/2),x)
[Out] -(x - 3)/(9*(4*x^2 - 24*x + 27)^(1/2))
```

3.130 $\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{5-2x}{9\sqrt{5-4x-x^2}}$$

```
[Out] 1/9*(5-2*x)/(-x^2-4*x+5)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {650}

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(5 - 4*x - x^2)^(3/2),x]
```

```
[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{(5-4x-x^2)^{3/2}}\,dx = \frac{5-2x}{9\sqrt{5-4x-x^2}}$$

Mathematica [A]
time = 0.08, size = 33, normalized size = 1.43

$$\frac{(-5+2x)\sqrt{5-4x-x^2}}{9(-1+x)(5+x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(5 - 4*x - x^2)^(3/2),x]
```

```
[Out] ((-5 + 2*x)*Sqrt[5 - 4*x - x^2])/(9*(-1 + x)*(5 + x))
```

Maple [A]
time = 0.50, size = 33, normalized size = 1.43

method	result	size
risch	$-\frac{2x-5}{9\sqrt{-x^2-4x+5}}$	20
gosper	$\frac{(x+5)(x-1)(2x-5)}{9(-x^2-4x+5)^{\frac{3}{2}}}$	26
trager	$\frac{(2x-5)\sqrt{-x^2-4x+5}}{9x^2+36x-45}$	30
default	$\frac{1}{\sqrt{-x^2-4x+5}}+\frac{-2x-4}{9\sqrt{-x^2-4x+5}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2-4*x+5)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/(-x^2-4*x+5)^(1/2)+1/9*(-2*x-4)/(-x^2-4*x+5)^(1/2)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.30

$$-\frac{2\,x}{9\,\sqrt{-x^2-4\,x+5}}+\frac{5}{9\,\sqrt{-x^2-4\,x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="maxima")
[Out] -2/9*x/sqrt(-x^2 - 4*x + 5) + 5/9/sqrt(-x^2 - 4*x + 5)
```

Fricas [A]
time = 1.29, size = 29, normalized size = 1.26

$$\frac{\sqrt{-x^2-4\,x+5}\,(2\,x-5)}{9\,(x^2+4\,x-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="fricas")
[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{(-(x-1)\,(x+5))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2-4*x+5)**(3/2),x)
```

```
[Out] Integral(x/(-(x - 1)*(x + 5))**(3/2), x)
```

Giac [A]
time = 1.63, size = 29, normalized size = 1.26

$$\frac{\sqrt{-x^2-4\,x+5}\,(2\,x-5)}{9\,(x^2+4\,x-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 0.83

$$-\frac{2\,x-5}{9\,\sqrt{-x^2-4\,x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5 - x^2 - 4*x)^(3/2),x)
```

```
[Out] -(2*x - 5)/(9*(5 - x^2 - 4*x)^(1/2))
```

3.131 $\int \frac{1}{(5-4x-x^2)^{5/2}} \, dx$

Optimal. Leaf size=43

$$\frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2(2+x)}{243\sqrt{5-4x-x^2}}$$

```
[Out] 1/27*(2+x)/(-x^2-4*x+5)^(3/2)+2/243*(2+x)/(-x^2-4*x+5)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {628, 627}

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(5-4x-x^2\right)^{5/2}}\,dx &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2}{27}\int \frac{1}{\left(5-4x-x^2\right)^{3/2}}\,dx\\ &= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}}+\frac{2(2+x)}{243\sqrt{5-4x-x^2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 43, normalized size = 1.00

$$\frac{\sqrt{5-4x-x^2}\,(38+3x-12x^2-2x^3)}{243(-1+x)^2(5+x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] (Sqrt[5 - 4*x - x^2]*(38 + 3*x - 12*x^2 - 2*x^3))/(243*(-1 + x)^2*(5 + x)^2
)
```

Maple [A]
time = 0.49, size = 40, normalized size = 0.93

method	result	size
gosper	$\frac{(x+5)(x-1)(2x^3+12x^2-3x-38)}{243(-x^2-4x+5)^{\frac{5}{2}}}$	36
default	$-\frac{-2x-4}{54(-x^2-4x+5)^{\frac{3}{2}}}-\frac{-2x-4}{243\sqrt{-x^2-4x+5}}$	40
trager	$-\frac{(2x^3+12x^2-3x-38)\sqrt{-x^2-4x+5}}{243(x^2+4x-5)^2}$	40
risch	$\frac{2x^3+12x^2-3x-38}{243(x^2+4x-5)\sqrt{-x^2-4x+5}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-4*x+5)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(-2*x-4)/(-x^2-4*x+5)^(3/2)-1/243*(-2*x-4)/(-x^2-4*x+5)^(1/2)
```

Maxima [A]
time = 0.28, size = 59, normalized size = 1.37

$$\frac{2\,x}{243\,\sqrt{-x^2-4\,x+5}}+\frac{4}{243\,\sqrt{-x^2-4\,x+5}}+\frac{x}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}}+\frac{2}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 -
4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

Fricas [A]
time = 2.10, size = 49, normalized size = 1.14

$$-\frac{(2\,x^3+12\,x^2-3\,x-38)\sqrt{-x^2-4\,x+5}}{243\,(x^4+8\,x^3+6\,x^2-40\,x+25)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x^
2 - 40*x + 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{2} - 4x + 5\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

Giac [A]
time = 1.63, size = 36, normalized size = 0.84

$$-\frac{\left(\left(2\,\left(x+6\right)x-3\right)x-38\right)\sqrt{-x^2-4\,x+5}}{243\,\left(x^2+4\,x-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="giac")
```

```
[Out] -1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x - 5)^2
```

Mupad [B]
time = 0.03, size = 29, normalized size = 0.67

$$-\frac{\left(4\,x+8\right)\,\left(8\,x^2+32\,x-76\right)}{3888\,{\left(-x^2-4\,x+5\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5 - x^2 - 4*x)^(5/2),x)
```

```
[Out] -((4*x + 8)*(32*x + 8*x^2 - 76))/(3888*(5 - x^2 - 4*x)^(3/2))
```

3.132 $\int (a+bx+cx^2)^p\,dx$

Optimal. Leaf size=122

$$-\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx+cx^2)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(1+p)}$$

```
[Out] -2^(1+p)*(c*x^2+b*x+a)^(1+p)*hypergeom([-p, 1+p],[2+p],1/2*(b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(-1-p)/(1+p)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {638}

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx+cx^2)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^p,x]
```

```
[Out] -((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p
)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int (a+bx+cx^2)^p\,dx=-\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx+cx^2)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(1+p)}$$

Mathematica [A]
time = 0.08, size = 126, normalized size = 1.03

$$\frac{2^{-1+p}\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\,{}_2F_1\left(-p,1+p;2+p;\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right)}{c(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(a + x*(b + c*x))^p*Hypergeomet
ric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a
*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(c\,x^{2}+b x +a \right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^p,x)
```

```
[Out] int((c*x^2+b*x+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx + cx^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**p,x)
```

```
[Out] Integral((a + b*x + c*x**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x^2+b\,x+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^p,x)
```

```
[Out] int((a + b*x + c*x^2)^p, x)
```

3.133 $\int (3+4x+5x^2)^p \, dx$

Optimal. Leaf size=37

$$5^{-1-p}11^p(2+5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(2+5x)^2\right)$$

```
[Out] 5^(-1-p)*11^p*(2+5*x)*hypergeom([1/2, -p],[3/2],-1/11*(2+5*x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$5^{-p-1}11^p(5x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(5x+2)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + 5*x^2)^p,x]
```

```
[Out] 5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/11*(2 + 5*x)^2
]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int (3+4x+5x^2)^p \, dx &= \frac{1}{2}\left(5^{-1-p}11^p\right)\text{Subst}\left(\int \left(1+\frac{x^2}{44}\right)^p \, dx, x, 4+10x\right)\\ &= 5^{-1-p}11^p(2+5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(2+5x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$5^{-1-p}11^p(2+5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(2+5x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + 5*x^2)^p,x]
[Out] 5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/11*(2 + 5*x)^2
]
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(5x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+4*x+3)^p,x)
[Out] int((5*x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+4*x+3)^p,x, algorithm="maxima")
[Out] integrate((5*x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+4*x+3)^p,x, algorithm="fricas")
[Out] integral((5*x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(5x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+4*x+3)**p,x)
```

```
[Out] Integral((5*x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(5\,x^2+4\,x+3\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 5*x^2 + 3)^p,x)
```

```
[Out] int((4*x + 5*x^2 + 3)^p, x)
```

3.134 $\int (3+4x+4x^2)^p \, dx$

Optimal. Leaf size=32

$$2^{-1+p}(1+2x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{2}(1+2x)^2\right)$$

```
[Out] 2^(-1+p)*(1+2*x)*hypergeom([1/2, -p],[3/2],-1/2*(1+2*x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$2^{p-1}(2x+1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{2}(2x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + 4*x^2)^p,x]
```

```
[Out] 2^(-1 + p)*(1 + 2*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/2*(1 + 2*x)^2]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int (3+4x+4x^2)^p \, dx &= 2^{-3+p}\text{Subst}\left(\int \left(1+\frac{x^2}{32}\right)^p \, dx, x, 4+8x\right)\\ &= 2^{-1+p}(1+2x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{2}(1+2x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$2^{-3+p}(4+8x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{32}(4+8x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + 4*x^2)^p,x]
[Out] 2^(-3 + p)*(4 + 8*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/32*(4 + 8*x)^2]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(4x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+4*x+3)^p,x)
[Out] int((4*x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+4*x+3)^p,x, algorithm="maxima")
[Out] integrate((4*x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+4*x+3)^p,x, algorithm="fricas")
[Out] integral((4*x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+4*x+3)**p,x)
```

```
[Out] Integral((4*x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(4\,x^2+4\,x+3\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 3)^p,x)
```

```
[Out] int((4*x + 4*x^2 + 3)^p, x)
```

3.135 $\int (3+4x+3x^2)^p \, dx$

Optimal. Leaf size=37

$$3^{-1-p}5^p(2+3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(2+3x)^2\right)$$

```
[Out] 3^(-1-p)*5^p*(2+3*x)*hypergeom([1/2, -p],[3/2],-1/5*(2+3*x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$3^{-p-1}5^p(3x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(3x+2)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + 3*x^2)^p,x]
```

```
[Out] 3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/5*(2 + 3*x)^2]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(3+4x+3x^2\right)^p \, dx &= \frac{1}{2}\left(3^{-1-p}5^p\right)\text{Subst}\left(\int \left(1+\frac{x^2}{20}\right)^p \, dx, x, 4+6x\right)\\ &= 3^{-1-p}5^p(2+3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(2+3x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$3^{-1-p}5^p(2+3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(2+3x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + 3*x^2)^p,x]
[Out] 3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/5*(2 + 3*x)^2]
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(3x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+3)^p,x)
[Out] int((3*x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+3)^p,x, algorithm="maxima")
[Out] integrate((3*x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+3)^p,x, algorithm="fricas")
[Out] integral((3*x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+3)**p,x)
```

```
[Out] Integral((3*x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(3\,x^2+4\,x+3\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2 + 3)^p,x)
```

```
[Out] int((4*x + 3*x^2 + 3)^p, x)
```

3.136 $\int (3+4x+2x^2)^p \, dx$

Optimal. Leaf size=21

$$(1+x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(1+x)^2\right)$$

```
[Out] (1+x)*hypergeom([1/2, -p],[3/2],-2*(1+x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$(x+1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + 2*x^2)^p,x]
```

```
[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int (3+4x+2x^2)^p \, dx &= \frac{1}{4}\text{Subst}\left(\int \left(1+\frac{x^2}{8}\right)^p \, dx, x, 4+4x\right)\\ &= (1+x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(1+x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$(1+x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(1+x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + 2*x^2)^p,x]
[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(2x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+4*x+3)^p,x)
[Out] int((2*x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+4*x+3)^p,x, algorithm="maxima")
[Out] integrate((2*x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+4*x+3)^p,x, algorithm="fricas")
[Out] integral((2*x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+4*x+3)**p,x)
```

```
[Out] Integral((2*x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int {\left(2\,x^2+4\,x+3\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2*x^2 + 3)^p,x)
```

```
[Out] int((4*x + 2*x^2 + 3)^p, x)
```

3.137 $\int (3+4x+x^2)^p \, dx$

Optimal. Leaf size=54

$$-\frac{2^{1+2p}(-2-2x)^{-1-p}(3+4x+x^2)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{3+x}{2}\right)}{1+p}$$

```
[Out] -2^(1+2*p)*(-2-2*x)^(-1-p)*(x^2+4*x+3)^(1+p)*hypergeom([-p, 1+p],[2+p],3/2+
1/2*x)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {638}

$$-\frac{2^{2p+1}(-2x-2)^{-p-1}(x^2+4x+3)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{x+3}{2}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x + x^2)^p,x]
```

```
[Out] -((2^(1 + 2*p)*(-2 - 2*x)^(-1 - p)*(3 + 4*x + x^2)^(1 + p)*Hypergeometric2F
1[-p, 1 + p, 2 + p, (3 + x)/2])/(1 + p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int (3+4x+x^2)^p \, dx = -\frac{2^{1+2p}(-2-2x)^{-1-p}(3+4x+x^2)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{3+x}{2}\right)}{1+p}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 0.89

$$\frac{2^p(1+x)(3+x)^{-p}(3+4x+x^2)^p\,{}_2F_1\left(-p,1+p;2+p;\frac{1}{2}(-1-x)\right)}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x + x^2)^p,x]
```

```
[Out] (2^p*(1 + x)*(3 + 4*x + x^2)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-1 - x)
/2])/((1 + p)*(3 + x)^p)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(x^{2}+4x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x+3)^p,x)
```

```
[Out] int((x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{2}+4x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x+3)**p,x)
```

```
[Out] Integral((x**2 + 4*x + 3)**p, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x+3)^p,x, algorithm="giac")
[Out] integrate((x^2 + 4*x + 3)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(x^2+4\,x+3\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + x^2 + 3)^p,x)
[Out] int((4*x + x^2 + 3)^p, x)
```

3.138 $\int (3+4x)^p\,dx$

Optimal. Leaf size=18

$$\frac{(3+4x)^{1+p}}{4(1+p)}$$

```
[Out] 1/4*(3+4*x)^(1+p)/(1+p)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)^p,x]
```

```
[Out] (3 + 4*x)^(1 + p)/(4*(1 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (3+4x)^p\,dx = \frac{(3+4x)^{1+p}}{4(1+p)}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.94

$$\frac{(3+4x)^{1+p}}{4+4p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)^p,x]
```

```
[Out] (3 + 4*x)^(1 + p)/(4 + 4*p)
```

Maple [A]
time = 0.40, size = 17, normalized size = 0.94

method	result	size
gosper	$\frac{(3+4x)^{1+p}}{4p+4}$	17
default	$\frac{(3+4x)^{1+p}}{4p+4}$	17
meijerg	$3^p x \operatorname{hypergeom}\left([1,-p],[2],-\frac{4x}{3}\right)$	17
risch	$\frac{(3+4x)(3+4x)^p}{4p+4}$	20
norman	$\frac{x\,e^{p\ln(3+4x)}}{1+p}+\frac{3\,e^{p\ln(3+4x)}}{4(1+p)}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*x)^p,x,method=_RETURNVERBOSE)
[Out] 1/4*(3+4*x)^(1+p)/(1+p)
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.89

$$\frac{(4\,x+3)^{p+1}}{4\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="maxima")
[Out] 1/4*(4*x + 3)^(p + 1)/(p + 1)
```

Fricas [A]
time = 1.28, size = 19, normalized size = 1.06

$$\frac{(4\,x+3)^p(4\,x+3)}{4\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="fricas")
[Out] 1/4*(4*x + 3)^p*(4*x + 3)/(p + 1)
```

Sympy [A]
time = 0.01, size = 20, normalized size = 1.11

$$\frac{\begin{cases}\frac{(4x+3)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log(4x+3) & \text{otherwise}\end{cases}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)**p,x)
```

```
[Out] Piecewise(((4*x + 3)**(p + 1)/(p + 1), Ne(p, -1)), (log(4*x + 3), True))/4
```

Giac [A]
time = 3.32, size = 16, normalized size = 0.89

$$\frac{(4\,x+3)^{p+1}}{4\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)^p,x, algorithm="giac")
```

```
[Out] 1/4*(4*x + 3)^(p + 1)/(p + 1)
```

Mupad [B]
time = 0.39, size = 32, normalized size = 1.78

$$\begin{cases} \frac{\ln(4\,x+3)}{4} & \text{if } p=-1 \\ \frac{(4\,x+3)^{p+1}}{4\,(p+1)} & \text{if } p\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3)^p,x)
```

```
[Out] piecewise(p == -1, log(4*x + 3)/4, p ~= -1, (4*x + 3)^(p + 1)/(4*(p + 1)))
```

3.139 $\int (3+4x-x^2)^p \, dx$

Optimal. Leaf size=31

$$-7^p(2-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(2-x)^2\right)$$

```
[Out] -7^p*(2-x)*hypergeom([1/2, -p],[3/2],1/7*(2-x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$-7^p(2-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(2-x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - x^2)^p,x]
```

```
[Out] -(7^p*(2 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - x)^2/7])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int (3+4x-x^2)^p \, dx &= -\left(\frac{1}{2}7^p\text{Subst}\left(\int \left(1-\frac{x^2}{28}\right)^p \, dx,x,4-2x\right)\right)\\ &= -7^p(2-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(2-x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.84

$$7^p(-2+x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(-2+x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - x^2)^p,x]
```

```
[Out] 7^p*(-2 + x)*Hypergeometric2F1[1/2, -p, 3/2, (-2 + x)^2/7]
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(-x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+4*x+3)^p,x)
```

```
[Out] int((-x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4*x+3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4*x+3)^p,x, algorithm="fricas")
```

```
[Out] integral((-x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+4*x+3)**p,x)
```

```
[Out] Integral((-x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((-x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(-x^2+4\,x+3\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - x^2 + 3)^p,x)
```

```
[Out] int((4*x - x^2 + 3)^p, x)
```

3.140 $\int (3+4x-2x^2)^p \, dx$

Optimal. Leaf size=31

$$-5^p(1-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(1-x)^2\right)$$

```
[Out] -5^p*(1-x)*hypergeom([1/2, -p],[3/2],2/5*(1-x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$-5^p(1-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(1-x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - 2*x^2)^p,x]
```

```
[Out] -(5^p*(1 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(1 - x)^2)/5])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int (3+4x-2x^2)^p \, dx &= -\left(\frac{1}{4}5^p\text{Subst}\left(\int \left(1-\frac{x^2}{40}\right)^p \, dx, x, 4-4x\right)\right)\\ &= -5^p(1-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(1-x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.84

$$5^p(-1+x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(-1+x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - 2*x^2)^p,x]
[Out] 5^p*(-1 + x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(-1 + x)^2)/5]
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(-2x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+4*x+3)^p,x)
[Out] int((-2*x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="maxima")
[Out] integrate((-2*x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="fricas")
[Out] integral((-2*x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-2x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-2*x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((-2*x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (-2\,x^2+4\,x+3)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 2*x^2 + 3)^p,x)
```

```
[Out] int((4*x - 2*x^2 + 3)^p, x)
```

3.141 $\int (3+4x-3x^2)^p \, dx$

Optimal. Leaf size=38

$$-3^{-1-p}13^p(2-3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)$$

```
[Out] -3^(-1-p)*13^p*(2-3*x)*hypergeom([1/2, -p],[3/2],1/13*(2-3*x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$-3^{-p-1}13^p(2-3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - 3*x^2)^p,x]
```

```
[Out] -(3^(-1 - p)*13^p*(2 - 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 3*x)^2/13]
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int (3+4x-3x^2)^p \, dx &= -\left(\frac{1}{2}\left(3^{-1-p}13^p\right)\text{Subst}\left(\int \left(1-\frac{x^2}{52}\right)^p \, dx,x,4-6x\right)\right)\\ &= -3^{-1-p}13^p(2-3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.97

$$3^{-1-p}13^p(-2+3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - 3*x^2)^p,x]
[Out] 3^(-1 - p)*13^p*(-2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 3*x)^2/13]
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(-3x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4*x+3)^p,x)
[Out] int((-3*x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="maxima")
[Out] integrate((-3*x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="fricas")
[Out] integral((-3*x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-3x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-3*x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (-3\,x^2+4\,x+3)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 3*x^2 + 3)^p,x)
```

```
[Out] int((4*x - 3*x^2 + 3)^p, x)
```

3.142 $\int (3+4x-4x^2)^p \, dx$

Optimal. Leaf size=35

$$-2^{-1+2p}(1-2x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{4}(1-2x)^2\right)$$

```
[Out] -2^(-1+2*p)*(1-2*x)*hypergeom([1/2, -p],[3/2],1/4*(1-2*x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$-2^{2p-1}(1-2x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{4}(1-2x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - 4*x^2)^p,x]
```

```
[Out] -(2^(-1 + 2*p)*(1 - 2*x)*Hypergeometric2F1[1/2, -p, 3/2, (1 - 2*x)^2/4])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int (3+4x-4x^2)^p \, dx &= -\left(2^{-3+2p}\text{Subst}\left(\int \left(1-\frac{x^2}{64}\right)^p \, dx, x, 4-8x\right)\right)\\ &= -2^{-1+2p}(1-2x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{4}(1-2x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.00

$$-2^{-3+2p}(4-8x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{64}(4-8x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - 4*x^2)^p,x]
[Out] -(2^(-3 + 2*p)*(4 - 8*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 8*x)^2/64])
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(-4x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+4*x+3)^p,x)
[Out] int((-4*x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="maxima")
[Out] integrate((-4*x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="fricas")
[Out] integral((-4*x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-4x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-4*x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((-4*x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(-4\,x^2+4\,x+3\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 4*x^2 + 3)^p,x)
```

```
[Out] int((4*x - 4*x^2 + 3)^p, x)
```

3.143 $\int (3+4x-5x^2)^p \, dx$

Optimal. Leaf size=38

$$-5^{-1-p}19^p(2-5x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)$$

```
[Out] -5^(-1-p)*19^p*(2-5*x)*hypergeom([1/2, -p],[3/2],1/19*(2-5*x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 251}

$$-5^{-p-1}19^p(2-5x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x - 5*x^2)^p,x]
```

```
[Out] -(5^(-1 - p)*19^p*(2 - 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 5*x)^2/19]
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int (3+4x-5x^2)^p \, dx &= -\left(\frac{1}{2}(5^{-1-p}19^p)\,\text{Subst}\left(\int \left(1-\frac{x^2}{76}\right)^p dx, x, 4-10x\right)\right)\\ &= -5^{-1-p}19^p(2-5x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.97

$$5^{-1-p}19^p(-2+5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x - 5*x^2)^p,x]
[Out] 5^(-1 - p)*19^p*(-2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 5*x)^2/19]
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(-5x^2+4x+3\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5*x^2+4*x+3)^p,x)
[Out] int((-5*x^2+4*x+3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="maxima")
[Out] integrate((-5*x^2 + 4*x + 3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="fricas")
[Out] integral((-5*x^2 + 4*x + 3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-5x^2+4x+3\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-5*x**2 + 4*x + 3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="giac")
```

```
[Out] integrate((-5*x^2 + 4*x + 3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (-5\,x^2+4\,x+3)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 5*x^2 + 3)^p,x)
```

```
[Out] int((4*x - 5*x^2 + 3)^p, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```